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From monopoly to competition: When do optimal contests prevail? ☆

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ABSTRACT

We study competition among multiple contest designers in a general model. The goal of each contest designer is to maximize the sum of efforts of the contestants participating in their contest. Assuming symmetric contestants, our main result shows that the optimal contests in the monopolistic setting (i.e., those that maximize the sum of efforts in a model with a single contest designer) form an equilibrium in the model with competition. Under a very natural assumption, these contests are dominant, and the equilibrium that they form is unique. Moreover, the equilibria with the optimal contests are Pareto-optimal even when other equilibria emerge. In many natural cases, they also maximize the social welfare. Additional examples show that, with further generalizations of our model, optimal contests no longer prevail. Our results therefore highlight and clarify the borderline between settings in which optimal contests prevail and do not prevail.

1. Introduction

Many important economic and social interactions may be viewed as contests. The designer aims to maximize her abstract utility (e.g. workers' productivity, sales competitions, innovative ideas, information from contestants) by forming a contest, and contestants exert effort in hopes of winning a prize. The design of optimal contests is by now well understood in the monopolistic (single-contest) setting. For example, the contest designer may consider designing the "prize structure", splitting the total prize into multiple smaller prizes to reward multiple contestants. The literature provides a good understanding of the optimal prize structure. In many cases, a winner-takes-all contest is optimal in terms of maximizing either the sum of contestants' efforts or the single maximal effort (e.g. Barut and Kovenock, 1998; Kalra and Shi, 2001; Moldovanu and Sela, 2001; Terwiesch and Xu, 2008; Chawla et al., 2019).

While most of the existing literature on contest design focuses on a monopolistic contest with an exogenously given set of contestants, in reality, many times, there are multiple contests on a market and these contests must compete to attract contestants. This induces a *participation* vs. *effort* trade-off. Although the optimal contest in the single-contest setting induces maximal effort exertion

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from participating contestants, contestants are at the same time discouraged from choosing the more demanding contests. To attract contestants, a contest designer should design lucrative, easy contests that leave a large fraction of the total surplus to contestants. One might expect that a contest designer should carefully balance these two contradicting aspects (participation vs. effort extraction) when competing with other contests.

Previous literature has noticed the aforementioned trade-off but only few models formally studied it, most notably, Azmat and Möller (2009) and Stouras et al. (2020); see Section 1.2 for details.[1] Surprisingly, those papers suggested that either participation or effort may dominate the other, depending on the exact assumptions underlying their models. However, a clear-cut understanding of the borderline between the two cases is still out of sight as many simplifying assumptions were previously made and it is not clear which of these assumptions are necessary to tip the trade-off in one direction. The current paper therefore aims to provide a more general framework and analysis of competition among multiple contest designers. Our analysis suggests that a contestant-symmetric model is a key factor leading to the domination of effort over participation, even if allowing asymmetric contest designers and a rather general contest design space.

As two concrete applications, we consider cases where the designer may be able to either (i) influence the accuracy with which contestants' efforts are observed (or the level of noise in the contest), and/or (ii) determine the prize structure. An example for (i) is mentioned by Wang (2010): *"the International Table Tennis Federation (ITTF) changed the points scoring system for international matches from first to 21 to first to 11 in 2000. One reason for doing this is to reduce the accuracy level of the matches"*.[2] Examples for increasing effort observability include competitive ice-skating, soccer, chess (Hjort, 1994; Brams et al., 2023; Brams and Ismail, 2021). We give further details in Section vii of the online appendix. A realistic example for (ii) is the popular data-science contests platform, Kaggle, which allows contest designer to split the total prize to multiple smaller prizes (so that, e.g., second and third places will also receive a monetary award).[3] Our results give clear-cut implications for case (i) and meaningful but somewhat weaker implications for (ii); see further details below.

1.1. Overview of results and techniques

At a high level, our contest competition model is composed of three phases. In the first phase, every contest designer i chooses (and commits to) a contest from some general class of anonymous contests $\mathcal{S}_i$ available to i. In the second step, after seeing the contests chosen by the designers, each contestant chooses (possibly in a random way) one contest to participate in. Finally, in each contest, contestants invest efforts by playing a symmetric Nash equilibrium (which previous literature has shown to exist; see details in Section 2). Designers aim to maximize the sum of efforts exerted in their own contests and contestants aim to maximize the reward (or prize; we use these two words interchangeably) they receive minus their effort.

We identify two properties of contests and show as our first main result that if every contest designer chooses a contest that satisfies these two properties then we are at an equilibrium. The first property is called Monotonically Decreasing Utility (MDU), which simply says that a contestant's symmetric-equilibrium utility in the single contest game decreases as the number of contestants increases. The second property, called Maximal Rent Dissipation (MRD), formally defines the optimality of a contest – in the monopolistic case – among all contests in $\mathcal{S}_i$. A contest $C_i \in \mathcal{S}_i$ satisfies MRD if, for any other contest $C_i' \in \mathcal{S}_i$ and for any number of contestants k, a contestant's symmetric-equilibrium utility in the single contest C_i with k contestants is not larger than her symmetric-equilibrium utility in C_i' with k contestants. Thus, C_i minimizes the contestants' utilities and therefore maximizes the utility of the contest designer, among all contests available to the designer. In this sense, C_i is optimal for designer i in the single contest game. One example of an MRD contest is the all-pay auction. This is an especially useful example as an all-pay auction leaves zero expected utility to the contestants. It therefore satisfies the MRD property relative to *any* arbitrary set $\mathcal{S}_i$. We term such contests Full Rent Dissipation (FRD); see a discussion of this in Section 2. As far as we know, the equilibrium properties of competing all-pay auctions were previously discussed only in the limited model of Azmat and Möller (2009). MRD abstracts away from the specific contest format, highlighting the importance of rent dissipation to the equilibrium properties in the setting of competing contests. Furthermore, MDU and MRD are almost without loss of generality since we show that, given any existing set of contests $\mathcal{S}_i$, one can always construct an additional contest which satisfies MDU and MRD by combining contests in $\mathcal{S}_i$ (see details in Section 2.5).

Main results Our first main result (Theorem 3.1) is that contests satisfying both MDU and MRD form an equilibrium of the contest competition game, and that, in the case where designers can choose only MDU contests, MRD contests are weakly dominant. In the latter case, MRD contests are the only possible contests that emerge in equilibria (Theorem 3.3). If designers can choose non-MDU contests, non-MRD contests may emerge as additional equilibria (Example 3.2) on top of the equilibria that MRD contests form. However, we show that even when non-MRD contests form equilibria, choosing MRD contests is a Pareto-optimal outcome for contest designers (Theorem 3.7), maintaining the attractiveness of MRD contests to contest designers. In summary, effort indeed dominates participation in the aforementioned trade-off for the competing designers. We additionally show that MRD contests are welfare optimal in many natural cases (although not always; see Section 5). These conclusions hold regardless of the

[1] There is also a conceptually related literature on competing mechanisms. See a discussion in Section 1.2.3 on this literature and the fundamental differences from models of competing contests.

[2] "*The rationale for this is simple. The domination of China meant that there was little incentive for the other teams. Reducing the accuracy level increases the chance that a team other than China will win, thus inducing more effort from the other teams. This increased competition could in turn result in greater effort from the Chinese team*" (Wang, 2010).

[3] For example, AMEX ran a contest on Kaggle during May to August 2022 with a total prize of 100 K USD that was divided to four prizes for the first four places.

number of designers, the prizes they offer, and the classes of contests they can choose from, subject to the assumptions detailed below.

Important modeling assumptions Our model includes five important assumptions: (a) symmetric contestants who play symmetric strategies in equilibrium, (b) linear costs of effort, (c) complete information, (d) the prize is fully allocated, and (e) the total prize is exogenous. These assumptions are standard in the literature as we discuss in the literature review below. Furthermore, throughout the paper we examine the necessity of these assumptions to our results. We next give a high-level overview.

Most importantly, we assume that contestants are symmetric, in the sense that they have a symmetric cost of effort and that they play a symmetric equilibrium when choosing contests to participate in as well as investing efforts within their chosen contests. We give examples showing that with asymmetric contestants our results do not hold: it is no longer true that the optimal contests in the monopolistic setting continue to be optimal when there is competition among contests. Second, our analysis relies on the observation that the way we model any single contest – which is one of the standard models of single contests in the literature – implies that the social welfare generated by a contest is fixed and independent of effort: as long as $k \geq 1$ contestants participate in a contest, the social welfare is simply the total prize offered by the designer (the efforts cancel out in the social welfare summation). This is an implication of assuming a linear and symmetric cost of effort and assuming that the prize is valued in the same way by all contestants. The assumption that the prize is valued the same way by all contestants is natural in our setting, since the prize is monetary, but less natural in incomplete-information auction-like settings where the reward is a non-monetary object. For this reason, our results may not necessarily carry over to such private-values settings. Third, we assume that the prize is fully allocated to the contestants. In particular, if only one contestant shows up in a contest, that contestant receives the total prize for free. This assumption is natural in many types of contests, e.g., in R&D contests, in crowd-sourcing websites, in rewarding athletes, musicians, and actors, for their performance. Section 6 gives a more detailed discussion of our assumptions and their implications on our results, including the above issues. Our results therefore highlight and clarify the borderline between settings in which optimal contests prevail and settings in which this is not the case.

Intuition To see why the above assumptions imply our first main result, that MRD contests are at equilibrium in our contest competition game, consider the following reasoning. Suppose that designer i switches from some MRD contest C_i to a non-MRD contest C_i'. We first argue that in the new equilibrium outcome the sum of contestants utilities in contest i will weakly increase. This seems natural since C_i' extracts less surplus than C_i from the contestants in the single contest game. However, we need to be careful here as we have two contradicting effects. On the one hand, for a fixed number of contestants, C_i' indeed leaves a larger surplus than C_i to the contestants. On the other hand, this also implies that more contestants (in expectation) will choose to participate in contest i, decreasing the equilibrium utility of each contestant. Nevertheless, the fact that more contestants participate in contest i implies that less contestants participate in any other contest $j \neq i$. Therefore, the sum of contestants utilities in contest j increases since contest j has MDU. In equilibrium, a contestant's expected utility from choosing contest j must be equal to her expected utility from choosing contest i. Thus, the sum of contestants equilibrium utilities will increase in all contests, including in contest i. This is a type of a spillover effect. Now, since the social welfare in contest i is constant (irrespective of the contest chosen by i, as discussed above), the fact that the contestants' utilities increase implies that the utility of contest designer i decreases. Thus, the transition from C_i to a non-MRD contest C_i' can only decrease i's utility. To complete the picture, we provide examples showing that this conclusion does not hold if any of the above-mentioned assumptions is removed.

Of course, some details are overlooked by this high-level intuition. For example, strictly speaking, it is not true that the social welfare is constant, since with a certain probability no contestant shows up at contest i and in this case the resulting social welfare at contest i is zero. In addition, we also prove uniqueness and Pareto-optimality of equilibria outcomes with MRD contests. The full details, supporting examples, and further discussion are in Section 3 and Appendix B.

Applications We apply our general framework to competition among Tullock contests with varying prize structures. As corollaries, we obtain: (1) Choosing winner-takes-all contests, i.e., giving the entire reward to the winner, is an equilibrium for designers who can only adjust prize structures but not their observability of effort and/or quality, i.e., the Tullock parameter τ is exogenous (Corollary 4.3). (2) Choosing all-pay auctions is an equilibrium for designers who can only adjust observability but must choose winner-takes-all; in fact, we show that choosing any Tullock parameter $\tau \geq 2$ is an equilibrium (Corollary 4.2). (3) Choosing the winner-take-all contest with the largest possible observability is an equilibrium for designers that can adjust both prize structures and observability (Corollary 4.4).

1.2. Related literature

We review three strands of the literature. The first strand considers the optimality of contests in a monopolistic (single contest) setting, in terms of revenue for the designer, agent participation, etc. The second and third strands consider competition among contests and among mechanisms, respectively. The difference between these two strands is discussed. Our results are interesting in the way that they tie together the first two strands: We show that effort-maximizing contests (those that were identified in the first strand) are in an equilibrium in our competition model (that belongs to and follows the second strand). The takeaway message to a contest designer is that in case she is interested to maximize the sum of contestants' efforts, introducing competition does not change her basic goal of extracting maximal effort from a fixed set of contestants.

1.2.1. Optimal contests in the monopolistic setting

In the monopolistic setting, several works study optimal multiple-prize contest design intending to maximize the sum of efforts. Under different assumptions, most of these papers arrive at the same conclusion that the optimal contest is the winner-takes-all contest where the full prize is offered to the single contestant exerting the highest effort. For example, in all-pay auctions where contestants' efforts are fully observable, Barut and Kovenock (1998); Moldovanu and Sela (2001) show that a winner-takes-all all-pay auction is optimal assuming contestants have either linear or concave cost functions (interestingly, for convex cost functions their results vary). An exception is Glazer and Hassin (1988) who show that the optimal contest should offer equal prizes to all players except for the player with the lowest effort if players value the prize money by a strictly concave utility function. We consider linear utilities and linear cost functions as in Barut and Kovenock (1998) so the winner-takes-all is optimal in the monopolistic setting. Our results therefore show that winner-takes-all is a Pareto-optimal equilibrium (and possibly dominant) for the contest designers in the competitive setting as well. When contestants' efforts are not fully observable and their winning probabilities for different prizes are assumed to follow a Tullock success function, the optimal prize structure is once again a winner-takes-all (Clark and Riis, 1998). The winner-takes-all is also optimal in a stochastic-quality model (e.g., Kalra and Shi (2001); Ales et al. (2017)) where a contestant who exerts effort e_i produces a submission with random quality $Q_i = e_i + Z_i$ where Z_i follows some noise distribution, and Drugov and Ryvkin (2020) characterizes that this holds as long as the hazard-rate of the distribution is increasing. In this case, prizes are allocated based on the submission qualities. If the designer must offer a single prize, then the all-pay auction is optimal among all possible contests, as it induces full rent dissipation – contestants' utilities are reduced to zero and their sum of efforts is maximized (Baye et al., 1996). A line of the literature including Schweinzer and Segev (2012); Baye et al. (1996); Alcalde and Dahm (2010); Ewerhart (2017) studies symmetric equilibria and rent dissipation in optimal contests. Our formal results, which analyze contests that admit symmetric equilibria and certain dissipation properties, have concrete applications thanks to the existence and characterization results provided in these papers. The models of the latter three papers assume, as we do, linear costs of effort, complete information, that the prize is fully allocated, and exogenous total prize.

1.2.2. Competition among contests

Azmat and Möller (2009) study a model very similar to ours including our assumptions (a) - (e) discussed above. They suggest that, for contest designers maximizing the sum of efforts, the effort aspect is always dominating, and hence a single prize should be offered. However, it is not clear how robust this conclusion really is, since their model is restricted in two main aspects. First, it assumes that all contests have the same total prize to offer.[4] Second, and perhaps even more important, it restricts the choice of a contest and assumes that designers must choose a multiple-prize contest where contestants' winning probabilities for each prize are determined by a Tullock success function that is exogenous and identical for all contest designers. We significantly generalize this model in several aspects including these two issues: we allow for arbitrary and non-identical total prizes and we allow for a general contest design space. An example to the latter is the possibility to endogenize the observability parameter of a Tullock contest, as we next discuss.

An important appeal of a Tullock contest as a model for winner-determination in real-life contests is that it captures the fact that the quality of a submission is not fully observed by the contest designer (in some settings one would imagine that even the effort itself is not fully observed). Therefore, the actual vector of efforts $e_1, \dots, e_k$ of the k contestants in a certain contest (whether it is fully observed or not) cannot deterministically determine the winner, and winning is only probabilistic in the true efforts (Segev, 2020). The winning probability may be following a Tullock contest success function $e_i^\tau / (\sum_{j=1}^k e_j^\tau)$ using some parameter τ to capture such partial-observability by letting contestants with higher efforts win with higher probabilities (as τ increases, observability is better). With such a motivation, it seems that a contest designer always has the strategic option to artificially *reduce* her observability, e.g., using a Tullock contest success function with some parameter $\tau' < \tau$. Endogenizing the Tullock contest parameter was previously suggested in Michaels (1988); Nitzan (1994); Dasgupta and Nti (1998); Wang (2010); Polishchuk and Tonis (2013). In particular, in the tournament setting of O'keeffe et al. (1984), even given costless perfect monitoring, the optimal contest requires that effort be monitored imperfectly. Combining various prize structures with a limited choice of the parameter τ is a natural way to expand the set of possible contests to consider. This is not captured in previous models of competition among contests. Contest success functions may depend on the number of contestants in other complex ways and many additional examples of natural classes of contests exist (Corchón, 2007). Naturally, as the strategic flexibility of contest designers increases, existing outcomes may no longer be in equilibrium, and even if they do remain in equilibrium, additional more attractive equilibria might emerge. For example, a natural question is whether the strategic possibility to decrease observability implies that we see such a practice in reality. Our results in fact imply the opposite, that even if such a possibility exists, it is sub-optimal – a high-observability design is better.

DiPalantino and Vojnovic (2009) consider an incomplete information competition among contests. They focus on participation issues rather than on the strategic choices of contest designers, by assuming that all contests are all-pay. They explicitly characterize the relationship between contestants' participation behavior and contests' rewards, and find that rewards yield logarithmically diminishing returns with respect to participation levels. Körpeoğlu et al. (2017) consider an incomplete information contest model where contestants can participate in multiple contests, and contest designers use winner-takes-all contests while strategically choosing rewards to maximize the maximal submission quality minus reward. They show that, in several cases, contest designers benefit from contestants' participation in multiple contests.

[4] Their analysis seems to significantly rely on this assumption. It also formally assumes exactly two competing contests, although this aspect may be more easily generalized.

Stouras et al. (2024) study a setting with two contest designers designing prize structures to maximize the quality of the single best solution submitted by participants in their contests minus the payment to the participants. Contestants are symmetric and have a convex cost-of-effort function. They show that contest designers will choose different prize structures in the competition setting and in the monopolistic setting. The difference between their results and ours stems from two different modeling assumptions: (i) their cost of effort is non-linear while ours is linear and (ii) the utility of a contest designer is different. In fact, Section iv in the online appendix demonstrates that the assumption of linear costs of effort is necessary for our results. It shows a counter example to our results where we change only the cost of effort assumption and keep all other assumptions the same. We note that our model is more general than Stouras et al. (2024) in other aspects: our model allows any number of contest designers and general contest design spaces.

Deng et al. (2022) study computational aspects of a parallel participation model similar to ours but under incomplete information. They show that for a designer, given knowledge of all other designers' strategies, it is NP-hard to compute a best response, and show how to compute an approximately best response. When other designers' strategies are unknown, they construct a constant-ratio approximation of the worst-case best-response. They do not discuss equilibria outcomes in their model, while our complete-information model admits equilibria that we specifically characterize and that are easily computable.

1.2.3. Competing mechanisms

The literature on competing mechanisms studies a game very similar to the one considered here: multiple sellers each post a mechanism, buyers observe all mechanisms, and then each choose one to participate in (or, in equilibrium, mix between mechanisms). The aggregate conclusion of this literature (e.g., McAfee (1993); Levin and Smith (1994); Peters and Severinov (1997); Peters (1997, 2001); Albrecht et al. (2012)) is that competition induces more efficient outcomes and thus there is a contrast between the optimal mechanism in the monopolistic case (which is inefficient) versus the competition case (which is efficient). For example, Peters (2001) explicitly shows a trade-off between participation and surplus extraction, in his model. Despite the similarities between the competing mechanism game and our competing contest game, several fundamental differences in modeling assumptions explain why our conclusions significantly differ, as follows.

First, these classic results usually assume a "large market" with an infinite number of sellers, while we study a setting with a finite number of sellers (contest designers). Indeed, Burguet and Sákovics (1999); Pai (2009) studies a competing auction problem with a finite number of sellers and shows that, unlike the large market case, the sellers do not choose efficient auctions: they set strictly positive reservation prices and do not always allocate the good to the highest-value buyer. A related benefit of our discussion is that we can use standard game theoretic equilibrium notions, rather than specialized equilibria tailored for a large market that implies bounded rationality of the players, as in McAfee (1993); Peters and Severinov (1997).

Second, while the literature on competing mechanisms usually assumes incomplete information, we assume complete information and (most importantly) as a result all contestants have the same value for the reward which is the standard assumption for monetary rewards as is many times the case in contests. Hence, every contest is efficient as long as the reward is fully allocated. Although all efficient mechanisms are equivalent according to the celebrated "Revenue Equivalence Theorem", not all efficient contests are equivalent: two contests that both fully allocate the reward may generate different profits to the contest designer if they allocate the reward in different ways – for example, a contest that allocates the reward to the contestant with the largest effort with a higher probability generates more profit to the contest designer. Specifically, in this paper we restrict contest designers to choose efficient contests (i.e., contests that fully allocate the reward) and study which efficient contests are chosen in equilibrium. It turns out that those are still the profit-maximizing ones as in the monopolistic (single-contest) setting.

Some papers in the competing mechanism literature study a complete-information model with a finite number of buyers and sellers (as we do), e.g., Burdett et al. (2001); Galenianos et al. (2011); Julien et al. (2002). They restrict attention to posted-price selling mechanisms, which are natural in their context but probably less natural for contests. In contrast, we allow for a much more general class of contests. Another notable difference between their model and ours is the treatment in case one buyer shows up to a certain mechanism/contest. Their posted price mechanism may not allocate the item to the buyer while we assume the efficient alternative, namely that in this case the contestant/buyer wins the prize/item and exerts no effort. While both assumptions are reasonable in their respective contexts, the contrast between our results and the results of Burdett et al. (2001); Galenianos et al. (2011) demonstrates the importance of this case (i.e., only a single contestant shows up). In particular, under our assumption that sellers give the item for free if exactly one buyer arrives, our main result would have the sellers charge the optimal monopolistic price when more than one buyer arrives. The result of Burdett et al. (2001), which is qualitatively different, explicitly describes the equilibrium posted price for any number of buyers and sellers and shows that it is always strictly lower than the optimal monopolistic price. The model of Galenianos et al. (2011) allows asymmetric sellers, as we do. They show that the equilibrium outcome of their competing mechanism game is not welfare maximizing (where the welfare is the sum of all sellers' and buyers' utilities) unless sellers are symmetric. In contrast, in our competing contest model, the equilibrium outcome is always welfare-maximizing in several natural cases even with asymmetric sellers (see Section 5 for details). This again demonstrates the importance of the modeling assumption regarding the case of a single contestant in a contest/mechanism. See additional discussion regarding these issues in Section v of the online appendix.

2. Model and preliminaries

2.1. A single-contest game

A contest designer designs a contest among several contestants to maximize the sum of efforts exerted by the contestants in return for some reward to be divided among them according to some winning rule determined by the designer.

Formally, a contest C is composed of a reward (or a total prize) R and contest success functions (CSF) $\boldsymbol{f}^k : \mathbb{R}^k_{\geq 0} \to [0,1]^k$ for each number of contestants $k > 0$.[5] After observing the contest C and the number of contestants k, the contestants exert efforts $(e_1, \ldots, e_k) \in \mathbb{R}^k_{\geq 0}$ to compete for the reward. Each contestant ℓ receives a fraction $f^k_\ell(e_1, \ldots, e_k)$ of the reward, where $f^k_\ell(e_1, \ldots, e_k)$ is the ℓ-th coordinate of the vector $\boldsymbol{f}^k(e_1, \ldots, e_k)$. We allow general functions $f^k_\ell(\cdot)$ and only require that $\sum_{\ell=1}^k f^k_\ell(e_1, \ldots, e_k) \leq 1$. This captures a wide variety of contest success functions. For example, in a single-prize (or winner-takes-all) setting, $f^k_\ell(e_1, \ldots, e_k)$ is the probability that contestant ℓ wins the entire prize; Section 4.2 and Footnote 13 discuss how our model captures multi-prize settings. The utility of a contestant is the reward she gets minus the effort she exerts: $f^k_\ell(e_1, \ldots, e_k)R - e_\ell$. Altogether, for a given number of contestants k, this defines a complete-information game for the contestants.

Definition 2.1. A contest is *anonymous* if its contest success functions $\boldsymbol{f}^k : \mathbb{R}^k_{\geq 0} \to [0,1]^k$ satisfy, for any $k > 0$, for any $(e_1, \ldots, e_k) \in \mathbb{R}_{\geq 0}$ and any permutation π of $(1, \ldots, k)$,

$$\boldsymbol{f}^k\left(e_{\pi(1)}, \ldots, e_{\pi(k)}\right) = \left(f^k_{\pi(1)}(e_1, \ldots, e_k), \ldots, f^k_{\pi(k)}(e_1, \ldots, e_k)\right).$$

Definition 2.2. A contest *fully allocates the reward* if its CSF $\boldsymbol{f}^k : \mathbb{R}^k_{\geq 0} \to [0,1]^k$ satisfy, $\forall k > 0, \forall (e_1, \ldots, e_k) \in \mathbb{R}^k_{\geq 0}$, $\sum_{\ell=1}^k f^k_\ell(e_1, \ldots, e_k) = 1$.

Example 2.3. A Tullock contest (or more accurately, a single-prize Tullock contest) parameterized by $\tau \in [0, +\infty]$ has the following contest success function:

$$f^k_\ell(e_1, \ldots, e_k) = \begin{cases} \frac{e_\ell^\tau}{\sum_{j=1}^k e_j^\tau} & \text{if } e_j > 0 \text{ for some } j \in \{1, \ldots, k\}, \\ \frac{1}{k} & \text{otherwise.} \end{cases}$$

When $\tau \to +\infty$ the contest becomes an "All-Pay Auction (APA)" where the contestant with the highest effort wins with certainty (to maintain anonymity, if multiple contestants exert the highest effort, they all win with equal probability). A Tullock contest is anonymous and it fully allocates the reward.

The Tullock contest model captures a noisy mapping of effort to output via the randomness parameter τ. As τ increases the mapping becomes less noisy (more accurate). For example, suppose a contestant who exerts effort e_ℓ produces a random output $Y_\ell = \tau \log e_\ell + Z_\ell$ where Z_ℓ follows some noise distribution. Efforts are unobservable and the reward is allocated to the contestant with maximal Y_ℓ. In this setting, $f^k_\ell(e_1, ..., e_k)$ is the expected fraction of reward received by contestant ℓ, as a function of contestants' unobserved efforts. When the noise distribution is a Gumbel distribution, f becomes a Tullock function (Fu and Wu, 2019). Our model therefore captures some of the aspects of unobservable efforts and noisy mappings.

Definition 2.4. Let $\mathcal{C}_R$ be the set of all anonymous contests with reward R that fully allocate the reward and have a symmetric Nash equilibrium among k contestants $\forall k > 0$.

For example, Alcalde and Dahm (2010) and Baye et al. (1996) show that Tullock contests with parameters $\tau \in [0, \infty)$ and $\tau = \infty$ admit a symmetric Nash equilibrium (NE); thus, $\mathcal{C}_R$ contains all Tullock contests with reward R. Other examples of contests that admit symmetric NE are given in, e.g., the seminal works of Hirshleifer (1989); Nti (1997), a survey by Corchón (2007), as well as later works such as Amegashie (2012).

We assume throughout that all contestants in the same contest (with k contestants) play a symmetric NE of that contest. Formally, for every $C \in \mathcal{C}_R$ we fix a (mixed strategy) symmetric NE, i.e., $e_1, \ldots, e_\ell, \ldots, e_k$ are i.i.d. random variables that follow a distribution F defined by a mixed strategy NE. Since C is anonymous, in the symmetric NE all contestants get an equal expected fraction of the reward and hence their expected utilities are identical. We denote their identical expected utility by $\gamma_C(k) = \mathbb{E}_{e_1,\ldots,e_k \sim F}[f^k_\ell(e_1, \ldots, e_k)R - e_\ell]$. Moreover, since $C \in \mathcal{C}_R$ fully allocates the reward, we must have $\mathbb{E}_{e_1,\ldots,e_k \sim F}\left[f^k_\ell(e_1, \ldots, e_k)\right] = \frac{1}{k}$ and hence

[5] Notice that the reward R does not vary with the number of contestants. This is the common practice in, e.g., most common crowd-sourcing platforms where the contest designer posts the contest description including the reward before knowing the number of contestants. Also, note that we do allow the prize allocation method (as captured by $\boldsymbol{f}^k$) to vary with k. In full generality, our results hold even if the reward is weakly monotonically decreasing with k, i.e., the designer offers smaller rewards if more contestants arrive. This is not a well-studied case and we skip treating it formally for simplicity. Curiously, it can find interpretation when the designer has a cap over the number of contestants it can accommodate for a contest, e.g., NPR (2022) describes a story where a village's recreational contest became too popular and thus was canceled.

$$\gamma_C(k) = \frac{R}{k} - \mathbb{E}_{e_\ell \sim F}\left[e_\ell\right]. \tag{1}$$

If $k = 1$, the single contestant does not exert any effort, hence $\gamma_C(1) = R$. We also have $\gamma_C(k) \geq 0\ \forall k > 0$ since a contestant can choose to exert zero effort and guarantee non-negative utility. Moreover, since $e_\ell \geq 0$, $\gamma_C(k) \leq \frac{R}{k}$.

2.2. Contest competition game

We study a game where multiple contest designers compete by choosing their contest success functions. The total prize that each contest designer i offers is R_i, assumed to be an exogenous constant. Contestants observe these contests and each chooses one to participate in. The utility of contest designer i when the number of participants in her contest is $k \geq 1$ is defined to be the sum of contestants' efforts $\sum_{\ell=1}^{k} e_\ell$. When $k = 0$, designer i's utility is defined to be $\alpha_i \cdot R_i$, for some constant $0 \leq \alpha_i < 1$, since in this case the contest designer keeps the reward.[6] Using $\gamma_{C_i}(k)$, we can write the (expected) utility of the designer of a contest $C_i \in \mathcal{C}_{R_i}$ with $k \geq 1$ contestants by rearranging (1):

$$\mathbb{E}_{e_1,\ldots,e_k \sim F}\left[\sum_{\ell=1}^{k} e_\ell\right] = k\mathbb{E}_{e_\ell \sim F}\left[e_\ell\right] = R_i - k\gamma_{C_i}(k). \tag{2}$$

We assume that the utility of a contest designer is the expected sum of efforts even if this is sometimes not fully observable. This fits settings like workplace contests that aim to improve workers' productivity, when workers' productivity is linear. In an additive noise model, the expected sum of efforts is equal to the expected sum of qualities of submissions since the expected noise is usually assumed to be zero.

Definition 2.5. A complete-information *contest competition game* is denoted by $CCG(m, n, (R_i)_{i=1}^m, (\mathcal{S}_i)_{i=1}^m)$, where $m \geq 2$ is the number of contest designers, $n \geq 1$ is the number of contestants, $R_i > 0$ is the reward of contest i, and $\mathcal{S}_i \subseteq \mathcal{C}_{R_i}$. The game has two phases:

1. **Designers choose contests.** Each designer i chooses a contest $C_i \in \mathcal{S}_i$ simultaneously. Contestants observe the chosen contests $(C_1, \ldots, C_m)$.
2. **Contestants play a normal-form game of choosing in which contest to participate.** A pure strategy of each contestant in this game is to choose one contest. Importantly, contestants may play a mixed strategy: each contestant ℓ participates in each contest C_i $(i = 1, ..., m)$ with probability $p_{\ell i}$, $\sum_{i=1}^m p_{\ell i} = 1$. Let the probability distribution chosen by contestant ℓ be $\boldsymbol{p}_\ell = (p_{\ell 1}, ..., p_{\ell m})$.

After Nature assigns contestants to contests, utilities are realized as follows. If $k \geq 1$ contestants participate in contest C_i, each of them gains utility $\gamma_{C_i}(k)$ and designer i gains utility $R_i - k\gamma_{C_i}(k)$. If $k = 0$, the designer's utility is $\alpha_i \cdot R_i$.

An important element of our model is the space $\mathcal{S}_i$ of all possible contests a designer can *strategically* choose. We treat this as an abstract set of anonymous contests that fully allocate the reward and admit a symmetric Nash equilibrium among k contestants for any $k > 0$. A central example is the set of Tullock contests.

Contestants must participate in some contest, i.e., they cannot decide not to participate. This assumption is innocuous since, as remarked above, the contestants' utilities in equilibrium are always non-negative. (In fact, they are strictly positive since a contestant gets the reward for free when no other contestants show up, which happens with positive probability.) When a contestant decides on a level of effort to exert, she knows the total number of contestants k that participate in the same contest. In practice, contestants observe this number in physical contests (like sports contests) or if the designer reveals this information. Myerson and Wärneryd (2006) show that contest designers have an incentive to reveal the number of contestants because the expected aggregate effort in a contest with a commonly known number of contestants is higher in general; Lim and Matros (2009) show that for Tullock contests with binomial participation distribution, the designer always reveals the number k; Ryvkin and Drugov (2020) characterize that this is true in general for contests with concave marginal costs.

In the second phase, each contestant has a finite number m of possible pure strategies and the game is symmetric, hence there must exist at least one symmetric mixed strategy NE (Nash, 1951). We assume throughout that contestants play a symmetric equilibrium, so the probability distributions of all contestants are the same: $\boldsymbol{p}_1 = \boldsymbol{p}_2 = \cdots = \boldsymbol{p}_n$. Example ii.1 in the online appendix discusses the case where the contestants can choose an asymmetric equilibrium, where our main result may not hold. As argued by Burdett et al. (2001), a symmetric equilibrium requires no coordination among contestants and is more robust and natural than an asymmetric equilibrium. We denote by $\boldsymbol{p}(C_1, \ldots, C_m) \in \mathbb{R}^m$ the probability distribution chosen by the contestants at a symmetric equilibrium when the designers choose contests $(C_1, \ldots, C_m)$ in the first phase. If there are multiple symmetric equilibria, $\boldsymbol{p}(C_1, \ldots, C_m)$ can be any one of them; our results hold for all of them.[7]

Assume that designers choose contests $\boldsymbol{C} = (C_1, \ldots, C_m)$ and contestants choose symmetric participation probabilities $\boldsymbol{p}(\boldsymbol{C}) = (p_1, \ldots, p_m)$. For a contestant participating in C_i, the number of other contestants who also participate in C_i follows the binomial distribution $\mathrm{Bin}(n-1, p_i)$. Thus, the expected utility of a contestant participating in C_i, denoted by $\beta(C_i, p_i)$, is:

[6] When $\alpha_i \geq 1$, the contest designer prefers to opt out of the contest competition game and keep the reward. If the contest designer cannot do that then most of our results in fact hold for any $\alpha_i \geq 0$.

[7] In addition, Lemma 2.8 shows that the symmetric equilibrium is unique if a certain condition (satisfied by, e.g., all Tullock contests) holds.

$$\beta(C_i,p_i) = \mathbb{E}_{k\sim \mathrm{Bin}(n-1,p_i)}\left[\gamma_{C_i}(k+1)\right] = \sum_{k=0}^{n-1}\binom{n-1}{k}p_i^k(1-p_i)^{n-1-k}\gamma_{C_i}(k+1). \tag{3}$$

Let $\mathrm{Supp}(\boldsymbol{C}) = \{i : p_i(\boldsymbol{C}) > 0\}$ be the set of indices of contests in which contestants participate with positive probability (i.e., the support of $\boldsymbol{p}(\boldsymbol{C})$).

Claim 2.1 *(Equilibrium condition in the participation game of the contestants). Suppose that designers choose contests $\boldsymbol{C} = (C_1,\dots,C_m)$ in the first phase of the game and contestants participate in contests with probabilities $\boldsymbol{p}(\boldsymbol{C}) = (p_1,\dots,p_m)$ in equilibrium. Then,*

- *For $i \in \mathrm{Supp}(\boldsymbol{C})$, $\beta(C_i,p_i) \geq \beta(C_j,p_j)$, $\forall j = 1,\dots,m$.*
- *Thus, for $i,j \in \mathrm{Supp}(\boldsymbol{C})$, $\beta(C_i,p_i) = \beta(C_j,p_j)$.*

Proof. $(p_1,\dots,p_m)$ is a symmetric equilibrium, i.e., given that all other contestants play $(p_1,\dots,p_m)$, a player's best response is to play $(p_1,\dots,p_m)$ herself. Therefore, the expected utility of choosing to participate in contest i is at least as high as choosing to participate in contest j, as contest i is assigned a positive probability $p_i > 0$. □

2.3. Equilibrium among contest designers

We use $\boldsymbol{C} = (C_i,\boldsymbol{C}_{-i}) = (C_1,\dots,C_m)$ to denote the contests (strategies) chosen by all designers, where $\boldsymbol{C}_{-i}$ denotes the contests chosen by designers other than i. Let $u_i(C_i,\boldsymbol{C}_{-i})$ be the expected utility of contest designer i given that contestants play the equilibrium $\boldsymbol{p}(C_i,\boldsymbol{C}_{-i})$. Formally, by (2) the utility of the designer of contest C_i equals $R_i - k\gamma_{C_i}(k)$ when there are $k \geq 1$ contestants. Since each contestant participates in C_i independently with probability $p_i = p_i(C_i,\boldsymbol{C}_{-i})$, the total number k of contestants participating in C_i follows the binomial distribution $\mathrm{Bin}(n,p_i)$, and hence the designer's expected utility equals

$$u_i(C_i,\boldsymbol{C}_{-i}) = \mathbb{E}_{k\sim \mathrm{Bin}(n,p_i)}\Big[\ \big(R_i - k\gamma_{C_i}(k)\big)\cdot \mathbb{1}_{[k\geq 1]}\ +\ \alpha_i R_i\cdot \mathbb{1}_{[k=0]}\ \Big]. \tag{4}$$

Claim 2.2. *The expected utility $u_i(C_i,\boldsymbol{C}_{-i})$ of designer i is equal to*

$$u_i(C_i,\boldsymbol{C}_{-i}) = \underbrace{R_i - (1-p_i)^n(1-\alpha_i)R_i}_{\text{expected welfare generated by contest } i} - \underbrace{np_i\beta(C_i,p_i).}_{\text{contestants' expected utility obtained from contest } i} \tag{5}$$

Proof.

$$\begin{aligned}
u_i(C_i,\boldsymbol{C}_{-i}) &= \mathbb{E}_{k\sim \mathrm{Bin}(n,p_i)}\Big[\big(R_i - k\gamma_{C_i}(k)\big)\cdot \mathbb{1}_{[k\geq 1]} + \alpha_i R_i\cdot \mathbb{1}_{[k=0]}\Big]\\
&= \sum_{k=1}^{n}\binom{n}{k}p_i^k(1-p_i)^{n-k}(R_i - k\gamma_{C_i}(k))\ +\ (1-p_i)^n\alpha_i R_i\\
&= \sum_{k=1}^{n}\binom{n}{k}p_i^k(1-p_i)^{n-k}R_i - \sum_{k=1}^{n}\binom{n}{k}p_i^k(1-p_i)^{n-k}k\gamma_{C_i}(k)\ +\ (1-p_i)^n\alpha_i R_i\\
&= R_i\left[1-(1-p_i)^n\right] + (1-p_i)^n\alpha_i R_i\ -\ \sum_{k=1}^{n}\binom{n}{k}p_i^k(1-p_i)^{n-k}k\gamma_{C_i}(k)\\
&= R_i - (1-p_i)^n(1-\alpha_i)R_i\ -\ np_i\sum_{k=1}^{n}\binom{n-1}{k-1}p_i^{k-1}(1-p_i)^{n-k}\gamma_{C_i}(k)\\
&= R_i - (1-p_i)^n(1-\alpha_i)R_i - np_i\mathbb{E}_{k'\sim \mathrm{Bin}(n-1,p_i)}\Big[\gamma_{C_i}(k'+1)\Big],
\end{aligned}$$

which equals $R_i - (1-p_i)^n(1-\alpha_i)R_i - np_i\beta(C_i,p_i)$ by (3). □

We analyze the following solution concepts:

Definition 2.6. Given some $CCG(m,n,(R_i)_{i=1}^m,(\mathcal{S}_i)_{i=1}^m)$,

- A contest $C_i \in \mathcal{S}_i$ is *(weakly) dominant* if $\forall C'_1 \in \mathcal{S}_1,\dots,C'_m \in \mathcal{S}_m$, $u_i(C_i,\boldsymbol{C'}_{-i}) \geq u_i(C'_i,\boldsymbol{C'}_{-i})$.
- A tuple of contests $(C_1,\dots,C_m)$, where $C_i \in \mathcal{S}_i$, is a *contestant-symmetric subgame-perfect equilibrium (contestant-symmetric SPE)* if $u_i(C_i,\boldsymbol{C}_{-i}) \geq u_i(C'_i,\boldsymbol{C}_{-i}), \forall C'_i \in \mathcal{S}_i, \forall i$.

For simplicity and also for practical purposes, we do not consider the case where designers play mixed strategies.

2.4. Some properties of contests

Our results use the following three properties of contests:

Definition 2.7.

- A contest $C_i \in \mathcal{C}_{R_i}$ has *monotonically decreasing utility (MDU)* if a contestant's symmetric NE utility is decreasing as the number of contestants increases: $\gamma_{C_i}(1) \geq \gamma_{C_i}(2) \geq \cdots \geq \gamma_{C_i}(n)$.
- A contest $C_i \in \mathcal{S}_i \subseteq \mathcal{C}_{R_i}$ has *maximal rent dissipation (MRD)* in $\mathcal{S}_i$ if $\forall C'_i \in \mathcal{S}_i$, $\forall k \in \{1, \ldots, n\}$, $\gamma_{C_i}(k) \leq \gamma_{C'_i}(k)$. In words, MRD contests minimize contestants' symmetric NE utility, or equivalently maximize the designer's utility, regardless of the number of contestants. Let $\mathrm{MRD}(\mathcal{S}_i) \subseteq \mathcal{S}_i$ denote the set of all MRD contests in $\mathcal{S}_i$.
- A contest $C_i \in \mathcal{C}_{R_i}$ has *full rent dissipation (FRD)* if $\gamma_{C_i}(1) = R_i$ and $\gamma_{C_i}(k) = 0$ for $k = 2, \ldots, n$.

Lemma 4.1 will show that all Tullock contests satisfy MDU. This is not unique to Tullock contests: e.g., Proposition 2 of Nti (1997) shows a large class of contests that also satisfy MDU. A full-rent-dissipation contest C_i satisfies both MDU and MRD in any set $\mathcal{S}_i$ that contains it. (Baye et al., 1996) show that APA has full rent dissipation. In fact, we note in Section 4.1 that every Tullock contest with $\tau \geq 2$ has full rent dissipation.

Arguably, one of the strengths of our approach is the level of abstraction of our contest model. For this reason, we embrace a characterization that follows the abstract MDU and MRD properties (as opposed to studying specific contest formats), though later we also consider specific applications and counter examples to MDU and MRD. Nevertheless, we point out that MDU and MRD are almost without loss of generality, since, given any existing set of contests $\mathcal{S}_i$, one can always construct an additional contest which satisfies MDU and MRD by combining contests in $\mathcal{S}_i$. This is formally discussed in Section 2.5.

We next briefly discuss some basic implications of MDU and MRD.

Claim 2.3. *If C_i has MDU and $p < p'$, then $\beta(C_i, p) > \beta(C_i, p')$.*

Proof. Appendix A. □

Claim 2.4. *Let $T_i \in \mathrm{MRD}(\mathcal{S}_i)$, and let $C_i \in \mathcal{S}_i$ be any other contest in $\mathcal{S}_i$. Then $\forall p \in [0, 1]$, $\beta(T_i, p) \leq \beta(C_i, p)$.*

Proof. By the definition of maximal rent dissipation contest, $\gamma_{C_i}(k+1) \geq \gamma_{T_i}(k+1)$ for all $k = 0, \ldots, n-1$, thus $\beta(C_i, p) = \mathbb{E}_{k \sim \mathrm{Bin}(n-1,p)}[\gamma_{C_i}(k+1)] \geq \mathbb{E}_{k \sim \mathrm{Bin}(n-1,p)}[\gamma_{T_i}(k+1)] = \beta(T_i, p)$. □

MDU also implies that the contestants have only one symmetric equilibrium when choosing contests to participate:

Lemma 2.8. *If $\boldsymbol{C} = (C_1, \ldots, C_m)$ are MDU contests, then the contestants' symmetric equilibrium $\boldsymbol{p}(\boldsymbol{C})$ is unique.*

Proof. Let $\boldsymbol{p} = (p_1, \ldots, p_m)$ and $\boldsymbol{p}' = (p'_1, \ldots, p'_m)$ be two different symmetric equilibria for contestants. Then there exist i, j such that $p_i > p'_i$ and $p_j < p'_j$ and we get the following contradiction:

$$\begin{aligned}
&\beta(C_i, p'_i) > && (p'_i < p_i, C_i \text{ has MDU, Claim 2.3})\\
&\beta(C_i, p_i) \geq && (p_i > 0, \text{Claim 2.1})\\
&\beta(C_j, p_j) > && (p_j < p'_j, C_j \text{ has MDU, Claim 2.3})\\
&\beta(C_j, p'_j) \geq && (p'_j > 0, \text{Claim 2.1})\\
&\beta(C_i, p'_i). && \square
\end{aligned}$$

2.5. Constructing an MDU and MRD contest from an existing $\mathcal{S}_i$

This subsection shows how, given any existing set of contests $\mathcal{S}_i$ (that might not have a MDU and MRD contest), one can always construct a new contest that satisfies MDU and MRD by combining the contests in $\mathcal{S}_i$. Thus, we believe that these two properties are almost without loss of generality. First, we point out that MRD is innocuous since the designer can let the contest depend on k (the number of participants): starting from any set of contests $\mathcal{S}_i$, we construct a new contest C_i^* that for each k selects the contest C_i^k that maximizes rent dissipation among the feasible set of contests for that specific k.[8] This contest C_i^* by definition has the MRD property with respect to the set $\mathcal{S}_i \cup \{C_i^*\}$. Then, for MDU (monotonically decreasing utility), while it is an arguably natural

[8] Formally, $C_i^k = \arg\min_{C_i \in \mathcal{S}_i} \gamma_{C_i}(k)$. We do assume that the "arg min" exists.

property of contests, we nevertheless show how to transform any non-MDU contest into an MDU contest without decreasing the contest designer's utility, using the following procedure:

MDU smoothing. Let C_i be a non-MDU contest. By definition, there exists an index k_1 such that $\gamma_{C_i}(k_1) < \gamma_{C_i}(k_1+1)$. Choose the minimal such k_1. We construct another contest C_i' as follows:

(1) When there are $k \neq k_1 + 1$ contestants participating, run the same contest as C_i.
(2) When there are $k_1 + 1$ contestants participating, first uniformly randomly sample a subset of k_1 contestants from the $k_1 + 1$ contestants, announce this subset, and then run the k_1-contestant contest C_i on this subset.
(3) Repeat this process until no such k_1 exists.

Note that, different from the single-contest model described in Section 2.1 where the contestants decide on efforts at the beginning of the contest, we assume here that contestants decide on efforts after seeing the realization of the random subset. This difference does not affect any of our results because our analysis only depends on (1) the contestants' equilibrium utility function $\gamma_{C_i}(\cdot)$, (2) anonymity of the contest, (3) the contest fully allocates the reward; We emphasize that our analysis does not depend on the specific contest format.

Lemma 2.9. *Let C_i be a non-MDU contest. The C_i' constructed by the MDU smoothing procedure satisfies MDU. Moreover, the contestants' utility is weakly smaller in C_i': $\forall k, \gamma_{C_i'}(k) \leq \gamma_{C_i}(k)$.*

Corollary 2.10. *If $C_i \in \mathrm{MRD}(\mathcal{S}_i)$ does not satisfy MDU, then by adding the C_i' in Lemma 2.9 to $\mathcal{S}_i$, we have $C_i' \in \mathrm{MRD}(\mathcal{S}_i \cup \{C_i'\})$ and C_i' satisfies MDU.*

Proof of Lemma 2.9. Consider any single iteration of MDU smoothing that transforms C_i to C_i'. Under C_i', when there are $k \neq k_1 + 1$ contestants, each contestant's equilibrium utility is the same as that in C_i:

$$\gamma_{C_i'}(k) = \gamma_{C_i}(k), \quad \forall k \neq k_1 + 1. \tag{6}$$

When there are $k_1 + 1$ contestants, each contestant will play the following strategy in symmetric equilibrium:

- If the contestant is in the subset of k_1 contestants, they play according to the equilibrium strategy in C_i, obtaining utility $\gamma_{C_i}(k_1)$;
- If the contestant is not in the subset of k_1 contestants, they exert 0 effort (because they will not be allocated the reward regardless of effort), obtaining utility 0.

So, the expected utility of a contestant in the symmetric equilibrium is:

$$\gamma_{C_i'}(k_1+1) = \Pr[\text{in the subset}] \cdot \gamma_{C_i}(k_1) + 0 = \frac{k_1}{k_1+1}\gamma_{C_i}(k_1) \ \leq \ \gamma_{C_i}(k_1) = \gamma_{C_i'}(k_1). \tag{7}$$

We see that $\gamma_{C_i'}(k_1+1) \leq \gamma_{C_i'}(k_1)$. This means that, by repeating the MDU smoothing procedure for all k_1 where $\gamma_{C_i}(k_1) < \gamma_{C_i}(k_1+1)$, we can turn C_i into a contest C_i' with monotonically decreasing utility function $\gamma_{C_i'}(k)$. Moreover, we note that the above construction ensures

$$\gamma_{C_i'}(k) \leq \gamma_{C_i}(k), \tag{8}$$

so the lemma is proved. □

3. Main results: equilibria in contest competition games

Our first main result shows that choosing MRD and MDU contests form a subgame perfect equilibrium of the CCG game. Moreover, MRD contests are dominant if the set of all possible contests contains only MDU contests:

Theorem 3.1.

1. *Fix any $CCG(m, n, (R_i)_{i=1}^m, (\mathcal{S}_i)_{i=1}^m)$ where each $\mathcal{S}_i \subseteq \mathcal{C}_{R_i}$ contains a maximal rent dissipation contest that has monotonically decreasing utility, denoted by $T_i \in \mathrm{MRD}(\mathcal{S}_i)$. Then, $(T_1, \dots, T_m)$ is a contestant-symmetric SPE.*
2. *Moreover, if each $\mathcal{S}_i$ contains only MDU contests, T_i is dominant for all i.*
3. *Corollary of 1: For any $CCG(m, n, (R_i)_{i=1}^m, (\mathcal{S}_i)_{i=1}^m)$ where each $\mathcal{S}_i \subseteq \mathcal{C}_{R_i}$ contains a full rent dissipation contest (e.g.,* APA*) denoted by F_i, $(F_1, \dots, F_m)$ is a contestant-symmetric SPE.*

We remark that if there are multiple contests with FRD or MRD properties, then these are "payoff equivalent" from both the contestants' and the designers' point of view. In other words, all players obtain the same expected utility no matter which contests are chosen in equilibrium.

The full proof of Theorem 3.1, as well as most other proofs in this section, are deferred to Appendix B. At a very high level, the argument for why MRD contests constitute an equilibrium for the contest competition game is the following:

Proof sketch. Fix a contest designer i. Suppose each of the n contestants participates in i's contest with some probability p_i (assuming a symmetric participation equilibrium). By Claim 2.2, i's expected utility is equal to

$$u_i(C_i, \boldsymbol{C}_{-i}) = R_i\left[1-(1-p_i)^n(1-\alpha_i)\right] - np_i\beta(C_i, p_i), \tag{9}$$

where we recall that $R_i\left[1-(1-p_i)^n(1-\alpha_i)\right]$ is the expected welfare generated in contest C_i and $\beta(C_i, p_i)$ is each contestant's expected utility conditioning on participating in C_i. Now, suppose that contest designer i switches to a contest C_i' that requires less effort from the contestants (namely, leaving more utility to the contestants) and hence increases the participation probability to $p_i' = p_i + \Delta p$. The welfare term $R_i\left[1-(1-p_i)^n(1-\alpha_i)\right]$ is increased by

$$\Delta p \cdot \frac{\partial}{\partial p_i} R_i\left[1-(1-p_i)^n(1-\alpha_i)\right] = nR_i(1-p_i)^{n-1}(1-\alpha_i)\Delta p. \tag{10}$$

A contestant's conditional utility in contest i is increased, because as contestants participate in *other* contests C_j $(j \neq i)$ with *lower* probabilities, the utility that a contestant obtains from C_j is *increased* because C_j has MDU by assumption; since contestants are indifferent between contests i and j, their utility obtained from contest i must be increased as well. Suppose it is increased to $\beta(C_i', p_i') = \beta(C_i, p_i) + \Delta\beta$. Then, the contestants' utility term $np_i\beta(C_j, p_i)$ in (9) is increased by

$$\begin{aligned} np_i'\beta(C_i', p_i') - np_i\beta(C_i, p_i) &= n(p_i + \Delta p)(\beta(C_i, p_i) + \Delta\beta) - np_i\beta(C_i, p_i) \\ &= n\beta(C_i, p_i)\Delta p + np_i\Delta\beta + n\Delta p\Delta\beta \\ &> n\beta(C_i, p_i)\Delta p \quad \geq \quad nR_i(1-p_i)^{n-1}\Delta p, \end{aligned} \tag{11}$$

where the last inequality $\beta(C_i, p_i) \geq R_i(1-p_i)^{n-1}$ is because a contestant obtains utility R_i when no other contestants participate in C_i, which happens with probability $(1-p_i)^{n-1}$. The amount of increase of contestants' utility (11) outweighs the increase of the welfare term (10), so the designer's total utility (9) is decreased. □

Theorem 3.1 gives a sufficient condition for the existence of a specific type of contestant-symmetric subgame-perfect equilibria, namely that the $\mathcal{S}_i$ contains MDU and MRD contests. Monotonically Decreasing Utilities rule out some design instruments, for example, caps on efforts (one could imagine a design in which the cap is decreasing in the number of contestants). We emphasize that, in our model, the $\mathcal{S}_i$ can contain other arbitrary types of contests, including non-MDU contests; the point is that if the $\mathcal{S}_i$ contains MDU and MRD contests then these contests form contestant-symmetric subgame-perfect equilibria. Nevertheless, by introducing caps on efforts (thus violating MDU), the following example shows that other types of equilibria may also exist and that MDU and MRD contests are not dominant. Section i in the online appendix gives additional examples and a more detailed discussion.

Example 3.2. Let $m = 2, n = 6$, $R_1 = R_2 = 1$, $\alpha_1 = \alpha_2 = 0$, both $\mathcal{S}_1$ and $\mathcal{S}_2$ consist of two contests: APA, and a contest C that gives the reward for free when the number of contestants is $k = 5, 6$ and runs APA otherwise. Thus, $\gamma_C = (1, 0, 0, 0, 1/5, 1/6)$, which is not MDU. The next paragraph shows that (C, C) is a contestant-symmetric SPE and that APA is not a best-response to C (and therefore not dominant). By Theorem 3.1, (APA, APA) is still a contestant-symmetric SPE of this game. Furthermore, by Theorem 3.7, both designers strictly prefer (APA, APA) over (C, C).

When designers choose (C, C), by symmetry, contestants participate in either contest with equal probability $(0.5, 0.5)$. By direct computation (e.g., using (5)), the expected utility of each designers is $(0.7812, 0.7812)$. Now suppose designer 1 switches to APA. The probabilities $(p_1, p_2) = \boldsymbol{p}(\text{APA}, C)$ in the contestants' symmetric mixed strategy Nash equilibrium must satisfy, according to Claim 2.1, $\beta(\text{APA}, p_1) = \beta(C, p_2)$ (assuming $p_1, p_2 > 0$). By numerical methods, we find that $(p_1, p_2) = (0.4061, 0.5939)$. The expected utility of designers is then $(0.7761, 0.7323)$. Since $0.7761 < 0.7812$, designer 1 does not switch to APA. By symmetry, designer 2 does not switch to APA. Hence, (C, C) is an equilibrium, and APA is not a dominant contest.

In this example, for every k, $\boldsymbol{f}^k$ uses a Tullock contest with a parameter τ_k that depends on k (namely, $\tau_k = 0$ for $k = 5, 6$ and $+\infty$ otherwise). An example where we use $\tau_k = 1$ instead of $\tau_k = 0$ for some k could be constructed in a similar way.[9]

When the sets $\mathcal{S}_i$ contain only MDU contests, the contestant-symmetric subgame-perfect equilibria that Theorem 3.1 describes are the only possible equilibria:

Theorem 3.3. *Fix any $CCG(m, n, (R_i)_{i=1}^m, (\mathcal{S}_i)_{i=1}^m)$ where each $\mathcal{S}_i \subseteq \mathcal{C}_{R_i}$ contains only MDU contests. Assume $\text{MRD}(\mathcal{S}_i) \neq \emptyset$ for each i. Pick $T_i \in \text{MRD}(\mathcal{S}_i)$, and let $\tilde{p}_i = p_i(T_1, \ldots, T_m)$ be the probability a contestant participates in contest T_i in the equilibrium of contestants, and let*

[9] In particular, $m = 2, n = 10$, $R_1 = R_2 = 1$, both $\mathcal{S}_1$ and $\mathcal{S}_2$ consist of two contests: the APA contest and a contest C with $\gamma_C = (1, 0, 0, 0, 0, 0, 1/49, 1/64, 1/81, 1/100)$, that is, choosing Tullock contest with $\tau_k = 1$ when $7 \leq k \leq 10$ and $\tau_k = +\infty$ otherwise. Then (C, C) is a contestant-symmetric SPE, and APA is not a dominant contest for either designer. See Example i.1 in the online appendix for details. Moreover, Example i.2 in the online appendix shows that even if $\mathcal{S}_1$ contains only contests with monotonically decreasing utility, APA may not be a dominant contest for designer 1.

$P = \text{Supp}(\boldsymbol{T}) = \{i : \tilde{p}_i > 0\}$ *be the set of indices of contests in which contestants participate with positive probability when the contests are* $(T_1, \ldots, T_m)$. *Then*

1. *for any contestant-symmetric SPE* $(C_1, \ldots, C_m)$, $p_i(C_1, \ldots, C_m) = \tilde{p}_i$.
2. *if* $|P| \geq 2$, *then* $(C_1, \ldots, C_m) \in \mathcal{S}_1 \times \cdots \times \mathcal{S}_m$ *is a contestant-symmetric SPE if and only if* $C_i \in \text{MRD}(\mathcal{S}_i), \forall i \in P$.[10]
3. *if* $|P| = 1$, *let* $P = \{i_0\}$, *then* $(C_1, \ldots, C_m) \in \mathcal{S}_1 \times \cdots \times \mathcal{S}_m$ *is a contestant-symmetric SPE if and only if* $\gamma_{C_{i_0}}(n) = \gamma_{T_{i_0}}(n)$.[11]

In the symmetric-reward case we can show that $|P| = m$, which makes the statement shorter:

Corollary 3.4. *Suppose each* $\mathcal{S}_i$ *contains only MDU contests, has* $\text{MRD}(\mathcal{S}_i) \neq \emptyset$, *and* $R_1 = \cdots = R_m$ *(symmetric rewards). Then* $(C_1, \ldots, C_m) \in \mathcal{S}_1 \times \cdots \times \mathcal{S}_m$ *is a contestant-symmetric SPE if and only if* $C_i \in \text{MRD}(\mathcal{S}_i)$ *for all* $i \in \{1, \ldots, m\}$.

Proof. Relying on the second item of Theorem 3.3, we only need to show that $|P| = m$ in this symmetric-reward case. Assume by contradiction that there exists i such that $\tilde{p}_i = 0$. Since $\sum_{\ell=1}^{m} \tilde{p}_\ell = 1$, there exists $j \neq i$ such that $\tilde{p}_j > 0$. Then by Claim 2.3, $\beta(T_j, \tilde{p}_j) < \beta(T_j, 0) = R_j = R_i = \beta(T_i, \tilde{p}_i)$. However, this contradicts the equilibrium condition (Claim 2.1) which states that $\tilde{p}_j > 0$ implies $\beta(T_j, \tilde{p}_j) \geq \beta(T_i, \tilde{p}_i)$. Therefore, we conclude that $\tilde{p}_i > 0$ for all $i \in \{1, \ldots, m\}$, i.e., $|P| = m$ as required. □

Thus, the case of symmetric rewards is a "clear cut" while the general case is more involved. The following example demonstrates the need for this distinction using a setting with highly asymmetric rewards. See Appendix B.4 for proof.

Example 3.5. Consider n contestants and $m \geq 3$ contest designers with $0 \leq \alpha_i < 1$. Contest 1 has reward $R_1 = 1$, and each other contest has reward $R_j = \left(\frac{m-1}{m-2}\right)^{n-1} + 1$. Each set $\mathcal{S}_i$ contains all MDU contests (hence contains APA). Then for any contest $C_1 \in \mathcal{S}_1$, $(C_1, T_2, \ldots, T_m)$ where $T_j = \text{APA} \in \text{MRD}(\mathcal{S}_j)$ for $j = 2, \ldots, m$ is a contestant-symmetric SPE. In this equilibrium, $p_1(C_1, T_2, \ldots, T_m) = 0$, and $p_j(C_1, T_2, \ldots, T_m) = \frac{1}{m-1} > 0$ for any $j = 2, \ldots, m$.

Finally, the equilibria in Theorem 3.1 are Pareto optimal for the contest designers:

Definition 3.6.

- For two strategy profiles $\hat{\boldsymbol{C}} = (\hat{C}_1, \ldots, \hat{C}_m), \boldsymbol{C} = (C_1, \ldots, C_m) \in \mathcal{S}_1 \times \cdots \times \mathcal{S}_m$ of the contest competition game $CCG(m, n, (R_i)_{i=1}^m, (\mathcal{S}_i)_{i=1}^m)$, we say $\boldsymbol{C}$ is a *Pareto improvement* of $\hat{\boldsymbol{C}}$, if $u_i(\boldsymbol{C}) \geq u_i(\hat{\boldsymbol{C}})$ for all $i \in \{1, \ldots, m\}$ and $u_i(\boldsymbol{C}) > u_i(\hat{\boldsymbol{C}})$ for at least one $i \in \{1, \ldots, m\}$.
- A strategy profile $\hat{\boldsymbol{C}} = (\hat{C}_1, \ldots, \hat{C}_m)$ is *Pareto Optimal* (PO) if there is no Pareto improvement of it.

Theorem 3.7. *The equilibria in Theorem 3.1 are PO.*

4. Applications

4.1. Competition among tullock contests

We first apply our results to the special case where the sets $\mathcal{S}_i$ are arbitrary subsets of Tullock contests, i.e., the parameter τ becomes a strategic choice (as suggested in e.g. Michaels (1988); Nitzan (1994); Wang (2010)). Some of the previous literature views τ as an exogenous parameter representing how accurately the designer can observe the ranking of contestants' efforts and the resulting qualities of their submissions. Even so, it seems plausible that the designer can choose an "ignorance is bliss" approach where she lowers the τ value (thus, observes efforts' ranking less accurately) to encourage participation. Such situations occurred in practice as we have discussed in the introduction.

It is known that APA has full rent dissipation (Baye et al., 1996) and in fact, as a corollary of Ewerhart (2017), every Tullock contest with parameter $\tau \geq 2$ has full rent dissipation. Thus, the class of Tullock contests contains maximal rent dissipation contests (namely, those with $\tau \geq 2$). Also, it is a class of contests that have monotonically decreasing utility:

Lemma 4.1 *(Corollary of Baye et al. (1996); Schweinzer and Segev (2012); Ewerhart (2017)). A contestant's utility in a Tullock contest* C_τ *with parameter* $\tau \in [0, +\infty]$, *reward* R, *and* k *total contestants equals*

[10] If $p_i(C_1, ..., C_m) = 0$, contest i could be anything since i's utility, which is 0, cannot be improved by choosing any other contest C_i' as $(C_i, \boldsymbol{C}_{-i})$ is an equilibrium. Moreover, by Claim B.1, $p_i(C_i', \boldsymbol{C}_{-i})$ must be 0 as well, so the choice of C_i' does not affect the choices of contests of other designers.

[11] As $p_{i_0}(C_1, \ldots, C_m) = 1$, with probability 1 there are n contestants in contest i_0, thus the contest success functions of contest i_0 for $k \neq n$ have no effect on the utility calculation for the contestants' best response and could be anything.

$$\gamma_{C_\tau}(k)=\begin{cases}R & \text{if } k=1,\\ R(\frac{1}{k}-\frac{k-1}{k^2}\tau) & \text{if } \frac{k}{k-1}\geq\tau,\\ 0 & \text{if } \frac{k}{k-1}<\tau\leq+\infty.\end{cases} \tag{12}$$

As corollaries,

- *Every Tullock contest satisfies MDU.*
- *Any Tullock contest with $\tau \geq 2$ has full rent dissipation.*
- *If $\mathcal{S}$ is the set of all Tullock contests with parameter τ in some range whose maximum $\tau^{\max}$ is well defined and at most 2, then the Tullock contest with $\tau^{\max}$ is the only contest in* MRD($\mathcal{S}$).

A proof of this lemma is given in Appendix C. Theorem 3.1 therefore immediately implies:

Corollary 4.2.

1. *Let $\mathcal{T}_{R_i}$ be the set of all Tullock contests with reward R_i. Then,* APA *and any other Tullock contest with $\tau \geq 2$ is a dominant contest for every designer in $CCG(m,n,(R_i)_{i=1}^m,(\mathcal{T}_{R_i})_{i=1}^m)$.*
2. *If $\mathcal{S}_i$ is the set of all Tullock contests with parameter τ_i in some range whose maximum $\tau_i^{\max}$ is well defined and at most 2. Then, the Tullock contest with $\tau_i^{\max}$ is the only dominant contest for every designer in $CCG(m,n,(R_i)_{i=1}^m,(\mathcal{S}_i)_{i=1}^m)$.*

4.2. Contests with varying prize structures

Clark and Riis (1998) consider a formal model of prize structures, as follows. Given a total prize R_i, each designer i divides it into multiple prizes $R_i^1>0,\dots,R_i^s>0$ after seeing the number of contestants k participating in her contest, with $\sum_{j=1}^s R_i^j=R_i$ and $s\leq k$.[12] Let e_ℓ denote the effort a contestant ℓ exerts in designer i's contest. Let $\tau>0$ be a fixed parameter. Each contestant in i's contest wins R_i^1 with probability proportional to $(e_\ell)^\tau$; after the winner of R_i^1 is determined, one of the remaining contestants is randomly chosen to win R_i^2 with probability again proportional to $(e_\ell)^\tau$; and so on.[13] As shown by Clark and Riis (1998), *as long as* pure-strategy symmetric effort-exerting equilibria for the contestants exist, setting $s=1$ and $R_i^1=R_i$ gives more utility to the designer than any other partition of the total reward. In our terminology, the contest with $s=1$, $R_i^1=R_i$ is an MRD contest. Thus, we obtain the following corollary:

Corollary 4.3. *Let $\mathcal{S}_i$ consist of multi-prize contests C_i under which a pure-strategy symmetric effort-exerting equilibria for the contestants exist. Then, each designer choosing the MRD contest where $s=1$ and $R_i^1=R_i$ is an equilibrium of the contest competition game.*

Clark and Riis (1998) show that a sufficient condition for the existence of pure-strategy symmetric effort-exerting equilibrium is $\tau\leq k/(k-1)$ and $s<(1-\frac{1}{e})k\approx 0.632k$. This means that the total prize is not split too finely (at most ≈63% of the contestants may win). Clark and Riis (1998)[page 613] explain: "The typical case where a symmetric equilibrium does not exist occurs when the prize-distribution has much weight at the end and at the beginning, and very little in the middle. The heavy prize mass at the end dampens the incentive to contribute, and may lead to a very low level of rent seeking". In other words, their sufficient conditions are meant to prevent extreme settings where low-quality submissions receive a relatively high prize (they describe, as an example, a case where there are ten contestants, nine prizes, and the last prize exceeds 22% of the total sum of prizes).

Indeed, such cases do not seem to be prevalent in reality. Even in cases where societal norms require awarding multiple prizes (e.g., contests for academic grants, promotions at work places), in reality, the number of prizes rarely exceeds 50% of the number of participants. This may be since it is infeasible or too costly for the designer to identify and rank all the losers, or simply because the same societal norms mandate an upper bound on the number of winners (e.g., an academic journal with an acceptance ratio of 63% will probably be perceived as a lower rank journal).

Regardless of the prize structure, a larger τ always gives a larger utility to a designer in the single-contest setting (Clark and Riis, 1998). Therefore, if both τ and the prize structures can be varied by the designers, incorporating the sufficient condition above, we obtain the following:

Corollary 4.4. *Let $\mathcal{S}_i$ consist of multi-prize contests C_i with adjustable parameter τ that satisfy: $0<\tau\leq n/(n-1)$, and for each $k\geq 2$ the number of divided prizes is at most $s<0.632k$. Then, each designer choosing the MRD contest where $s=1$, $R_i^1=R_i$, and τ is the largest possible parameter the designer can choose, is an equilibrium of the contest competition game.*

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[12] Azmat and Möller (2009) consider a similar model but assume that designers choose prize structures before knowing the number of contestants. We allow to condition the prize structure on the realized number of contestants, which is more general.

[13] This is a valid CSF in our model as $f_\ell^k(e_1,...,e_k)$ specifies the *expected fraction* of the total prize contestant ℓ wins. For the prize structures described here, $f_\ell^k(e_1,...,e_k)=(\pi_1 R_i^1+\cdots+\pi_s R_i^s)/R_i$ where π_j is the probability that ℓ wins the j'th prize.

5. Welfare optimality

Throughout this section, let $\boldsymbol{C} = (C_1, \ldots, C_m)$ be a tuple of contests. Denote the sum of designers' expected utilities (designers' welfare), the sum of contestants' expected utilities (contestants' welfare), and their sum (social welfare) by

$$W_D(\boldsymbol{C}) = \sum_{i=1}^{m} u_i(\boldsymbol{C}), \qquad W_C(\boldsymbol{C}) = n \sum_{i=1}^{m} p_i \beta(C_i, p_i), \qquad W_S(\boldsymbol{C}) = W_D(\boldsymbol{C}) + W_C(\boldsymbol{C})$$

where $p_i = p_i(\boldsymbol{C})$. By the equilibrium condition (Claim 2.1) we have $\beta(C_i, p_i) = \beta(C_j, p_j)$ for all $i, j \in \mathrm{Supp}(\boldsymbol{C})$. Denote this constant by $u_c(\boldsymbol{C})$. This is a contestant's expected utility in any contest in which she participates with positive probability. Note that by definition $p_i = 0$ for any $i \notin \mathrm{Supp}(\boldsymbol{C})$, so $\sum_{i \in \mathrm{Supp}(\boldsymbol{C})} p_i = \sum_{i=1}^{m} p_i = 1$. As a result,

$$W_C(\boldsymbol{C}) = n \sum_{i=1}^{m} p_i \beta(C_i, p_i) = n \sum_{i \in \mathrm{Supp}(\boldsymbol{C})} p_i \beta(C_i, p_i) = n \sum_{i \in \mathrm{Supp}(\boldsymbol{C})} p_i u_c(\boldsymbol{C}) = n u_c(\boldsymbol{C}).$$

For $W_S(\boldsymbol{C})$, note that whenever at least one contestant participates in contest i, the sum of expected utilities of designer i and the participants in that contest equals R_i. Define the random variables $k_1, \ldots, k_m$ as the number of contestants in contests $C_1, \ldots, C_m$. We can rewrite $W_S(\boldsymbol{C})$:

$$\begin{aligned}
W_S(\boldsymbol{C}) &= \mathbb{E}_{k_1,\ldots,k_m}\left[\sum_{i=1}^{m} R_i(\mathbb{1}_{[k_i \geq 1]} + \alpha_i \mathbb{1}_{[k_i = 0]})\right] \\
&= \sum_{i=1}^{m} R_i \mathbb{E}_{k_i \sim \mathrm{Bin}(n, p_i)}\left[\mathbb{1}_{[k_i \geq 1]} + \alpha_i \mathbb{1}_{[k_i = 0]}\right] \\
&= \sum_{i=1}^{m} R_i \mathbb{E}_{k_i \sim \mathrm{Bin}(n, p_i)}\left[1 - (1 - \alpha_i)\mathbb{1}_{[k_i = 0]}\right] \\
&= \sum_{i=1}^{m} R_i \left[1 - (1 - \alpha_i)(1 - p_i(\boldsymbol{C}))^n\right]. \qquad (13)
\end{aligned}$$

We show that the social welfare of the equilibria in Theorem 3.1 is optimal in several natural cases:

Theorem 5.1. *Consider a contest competition game $CCG(m, n, (R_i)_{i=1}^m, (\mathcal{S}_i)_{i=1}^m)$ with some $T_i \in \mathrm{MRD}(\mathcal{S}_i)$ that satisfies MDU. Assume that the contest designers value the reward in the same way: $\alpha_1 = \ldots = \alpha_m = \alpha \in [0, 1)$. Then the equilibrium $(T_1, \ldots, T_m)$ maximizes the social welfare W_S in each one of the following cases:*

1. *Unrestricted contest design: for all i, $\mathcal{S}_i = \mathcal{C}_{R_i}$.*
2. APA *is a possible contest:* $\forall i, \mathrm{APA} \in \mathcal{S}_i$. *(*APA *can be replaced with any other full rent dissipation contest.)*
3. *A symmetric CCG: $R_1 = \cdots = R_m = R$ and $\mathcal{S}_1 = \cdots = \mathcal{S}_m = \mathcal{S} \subseteq \mathcal{C}_R$.*
4. *An MRD-symmetric CCG: $R_1 = \cdots = R_m = R$ and $\mathrm{MRD}(\mathcal{S}_1) = \cdots = \mathrm{MRD}(\mathcal{S}_m)$.*

(The Case 2 generalizes Case 1, and Case 4 generalizes Case 3 since every symmetric CCG is also MRD-symmetric.) The proof of this theorem is in Appendix D. Example D.2 shows that these conclusions do not always hold outside of the four cases above. When contest designers value keeping the reward differently ($\alpha_1, \ldots, \alpha_m$ are different), Theorem 5.1 does not always hold either. For intuition, consider the case of two contests, two contestants, and equal rewards, $m = n = 2, R_1 = R_2 = R$. By Eq. (13), $W_S(\boldsymbol{C}) = 2R - (1 - \alpha_1) \cdot R \cdot (1 - p_1(\boldsymbol{C}))^2 - (1 - \alpha_2) \cdot R \cdot (1 - p_2(\boldsymbol{C}))^2$. The last two terms capture the loss in social welfare due to cases where both contestants participate in the same contest, while the other contest has no contestants. When $\alpha_1 < \alpha_2$ (designer 2 values keeping the reward more than designer 1), maximizing the social welfare requires the participation probabilities to satisfy $p_1(\boldsymbol{C}) > p_2(\boldsymbol{C})$, which means more contestants participate in contest 1 on average. But this may not necessarily be the case in the contestants' equilibrium, because the contestants' utilities are agnostic to the α_i values. Example D.4 shows a concrete example.

Although the total welfare is not always maximized at the equilibria in Theorem 3.1, the contestants' welfare is *always* minimized at those equilibria, as the next theorem shows.[14]

Theorem 5.2. *Fix some $CCG(m, n, (R_i)_{i=1}^m, (\mathcal{S}_i)_{i=1}^m)$. Let $\boldsymbol{T} = (T_1, \ldots, T_m)$ be one of the equilibria in Theorem 3.1, i.e., $T_i \in \mathrm{MRD}(\mathcal{S}_i)$ and T_i satisfying MDU. Then for any $\boldsymbol{C} \in \mathcal{S}_1 \times \cdots \times \mathcal{S}_m$, $W_C(\boldsymbol{T}) \leq W_C(\boldsymbol{C})$.*

Proof. Let $\tilde{p}_i = p_i(\boldsymbol{T})$ and $p_i = p_i(\boldsymbol{C})$. We assume without loss of generality $p_1 > 0$ (i.e. $1 \in \mathrm{Supp}(\boldsymbol{C})$). Then $u_c(\boldsymbol{C}) = \beta(C_1, p_1)$.

[14] Designers' PO does not immediately imply minimal contestants' welfare since (1) the game is not constant-sum as $W_S(\boldsymbol{C})$ depends on p_i's and (2) PO outcomes need not necessarily maximize the aggregate designers' utility.

If $p_1 \le \tilde{p}_1$, then $\tilde{p}_1 > 0$, which implies $u_c(\boldsymbol{T}) = \beta(T_1, \tilde{p}_1)$. Since T_1 has monotonically decreasing utility, we have $\beta(T_1, \tilde{p}_1) \le \beta(T_1, p_1)$ by Claim 2.3. Since T_1 is also the maximal rent dissipation contest of $\mathcal{S}_1$, we get $\beta(T_1, p_1) \le \beta(C_1, p_1)$ by Claim 2.4. These inequalities together yield

$$u_c(\boldsymbol{T}) = \beta(T_1, \tilde{p}_1) \le \beta(T_1, p_1) \le \beta(C_1, p_1) = u_c(\boldsymbol{C}).$$

Otherwise $p_1 > \tilde{p}_1$, then there exists $i \in \{2, \dots, m\}$ such that $\tilde{p}_i > p_i$. Note that this implies $\tilde{p}_i > 0$, so $u_c(\boldsymbol{T}) = \beta(T_i, \tilde{p}_i)$, and,

$$u_c(\boldsymbol{T}) = \beta(T_i, \tilde{p}_i) \overset{T_i \text{ has MDU, Claim 2.3}}{\le} \beta(T_i, p_i) \overset{T_i \in \mathrm{MRD}(\mathcal{S}_i), \text{ Claim 2.4}}{\le} \beta(C_i, p_i) \overset{p_1 > 0, \text{ Claim 2.1}}{\le} \beta(C_1, p_1) = u_c(\boldsymbol{C}).$$

In either case we would get $u_c(\boldsymbol{T}) \le u_c(\boldsymbol{C})$, hence $W_C(\boldsymbol{T}) = n u_c(\boldsymbol{T}) \le n u_c(\boldsymbol{C}) = W_C(\boldsymbol{C})$. □

Theorems 5.1 and 5.2 together immediately imply:

Corollary 5.3. *Consider a contest competition game $CCG(m, n, (R_i)_{i=1}^m, (\mathcal{S}_i)_{i=1}^m)$ with some $T_i \in \mathrm{MRD}(\mathcal{S}_i)$ satisfying MDU. The equilibrium $(T_1, \dots, T_m)$ maximizes the designers' welfare W_D under the assumptions of Theorem 5.1.*

Thus, for example, (APA, …, APA) maximizes the designers' welfare and minimizes the contestants' welfare in the case of unrestricted contest design, if contest designers value the reward in the same way.

6. Discussion

This paper studies a complete-information competition game among contest designers. First, each designer chooses a contest. Second, each contestant chooses (possibly in a random way) one contest to participate in. Third, a symmetric equilibrium outcome is realized in each contest (contestants choose effort levels; the reward is allocated to them based on their realized efforts). The resulting utility of each contest designer is the sum of efforts invested in their respective contests. The resulting utility of each contestant is the reward she receives minus the effort she invests.

Our main results characterize a certain type of contests that form an equilibrium (and may even be dominant) in this game. These equilibria are Pareto-optimal for the contest designers. Under natural conditions, these are the only possible equilibria. In addition, these equilibria maximize social welfare (while minimizing the contestants' aggregate welfare) when designers are unrestricted in their choice of a contest, or when they are all restricted in the same way. Our results yield several conclusions regarding Tullock contests. For example, if contest designers are restricted to choosing a Tullock contest with some parameter τ then any Tullock contest with $\tau \ge 2$ (e.g., APA where $\tau = \infty$) is a dominant contest for every designer.

The bottom line of our formal analysis is that under our assumptions a contest designer can ignore competition and focus solely on increasing the effort of the contestants that arrive to the contest. This conclusion advances the state-of-art as, a-priori, the contest competition game can yield outcomes that depend on the number of designers, contestants, and equilibrium selection. Game-theoretically, our conclusion is compelling as it is supported by Pareto-optimality and even by dominant strategies.

Clearly, each theory has its underlying assumptions and we aim to give the most general conditions under which our conclusions hold. In the following discussion, we explore which assumptions are necessary and which can be relaxed for the above to still hold. This gives both a prescription for designers as well as a basis on which further research to study alternative assumptions can develop. For example, we show that if the assumption of symmetric contestants is relaxed then our conclusions no longer hold. Examining and relaxing our assumptions are interesting future directions, as we next discuss.

The participation model: We discuss four assumptions. First, one may assume that contestants cannot observe the total number of contestants in the contest they chose which may be the reality in large electronic/online contests (but is less realistic in small physical contests). Second, a contestant may be able to participate in more than one contest simultaneously, e.g., in up to some fixed maximal number of contests. Another option is to study a budgeted participation model where the total effort of each contestant could be split among several contests (see e.g. Lavi and Shiran-Shvarzbard (2020)). Third, if we allow contestants to choose an asymmetric equilibrium in response to the designers' contest success functions, our results no longer hold. Example ii.1 in the online appendix shows that (APA, APA) is not an equilibrium when we have two Tullock contests and three contestants that may choose an asymmetric equilibrium for their participation probabilities. It can be interesting to understand the asymmetric case as well. Another interesting option is to consider an endogenous population of designers, e.g., Lazear and Rosen (1981) assumes infinitely many designers (in their setup, firms) implying that in a competitive equilibrium designer utility is zero.

Asymmetric and/or stochastic costs of effort: Our approach relies critically on the assumption that contestants have a symmetric and fixed unit cost of effort (normalized to 1). This enables symmetric equilibrium strategies for the contestants and also makes the single contest game a constant-sum game. The constant-sum property allows us to relate the designer's utility with the contestants' utilities which can then be characterized by the equilibrium condition for the contestants. When contestants have asymmetric fixed costs of effort or stochastic private costs of effort, the single contest game between a designer and $k \ge 1$ contestants is not constant-sum in general, so the argument in this paper cannot be applied directly. Another obstacle to analyzing asymmetric and/or stochastic costs of effort is the lack of explicit characterization of contestants' utilities in a contest (even in a single Tullock contest) in those settings. We give a concrete example in Section iii of the online appendix. It shows that the conclusion that APA contests form an equilibrium no longer holds when contestants have asymmetric costs of effort. The symmetry assumption is also an important ingredient in the literature on Tullock contests and all-pay auctions: e.g., neither of these two extracts full surplus with asymmetric contestants, even under complete information. Section vi in the online appendix gives an example where contestants have private stochastic costs of efforts and our conclusion that the optimal contests in the monopoly setting prevail in the competition setting still holds. This is a special example that satisfies the constant-sum property, so the argument in this paper applies. However, in general, the main argument in the example does not apply. Whether our conclusion holds in a more general incomplete-information setting is open.

Non-linear cost of effort: We assumed linear cost of effort for contestants. One main interesting direction to relax this assumption is non-linear cost of effort. Section iv of the online appendix shows an example where contestants have convex costs of effort and the (unique) MRD contest is no longer an equilibrium. Specifically, in the example, contestants have costs of effort $c(e) = e^b$ with $b = 3$. There are two contest designers that can choose some Tullock contest with $\tau \in [0, 6]$. Within this parameterized class of Tullock contests, $\tau = 6$ is the unique MRD contest. However, the outcome $(\tau = 6, \tau = 6)$ is no longer a contestant-symmetric SPE. It can be interesting to further investigate how contestants' costs of effort affect the qualitative conclusions of this work.

Non-MDU contests: When the prize structure can depend on the number of participants, e.g., choosing a different Tullock contest depending on the number of participants, the MDU property can be violated, see e.g., Example 3.2. This opens the possibility of new "exotic" contests with new types of equilibria that do not fall within Theorem 3.1. However, as we show in Section 2.5, it is rather straight-forward to extend an existing contests class to contain an MRD+MDU contest, using the existing contests' formats.

Reservation prices and the case of one contestant: Our results rely on the assumption that the reward is fully allocated and therefore allocated for free when a single contestant shows up at some contest. One may consider setting a threshold on minimal effort, but there are several issues with such an approach, as follows.

First, in online crowdsourcing contests, it is typical that the number of contestants is significantly larger than the number of contests.[15] In such a case, the probability that only one contestant shows up to a contest is very low. Therefore, this case seems insignificant to the contest designer. We conjecture that this case would not significantly affect our conclusion; this is supported by some preliminary analysis in the Appendix, see details below.

Second, when adding a threshold on effort, the contest designer needs to accurately observe the effort, which may not be always possible (this is unlike auctions where bids are monetary and fully observable). The difficulty of observing effort is in fact a key motivation in Lazear and Rosen (1981)'s rank-order tournament model. In reality, if the designer is not an expert it is hard for her to evaluate the effort invested in a submission. This seems especially true if there is only one contestant since in this case, a non-expert contest designer is not able to compare the submission to other submissions and therefore evaluating the effort it took to compose the submission is even harder. Thresholds on effort are not part of common contest design models. For example, in the Tullock contest model the prize must always be allocated regardless of the observability parameter. The difficulty in accurately evaluating effort, as described above, is one justification for that.

Third, we note that whether a contest designer can credibly commit to withholding the prize if the effort is lower than the threshold is questionable. If the contest designer decides not to hand out the full reward initially announced, *after* she received a submission, she may face some legal issues challenging her to prove that not enough effort was invested. In addition, the contest designer may be tempted to keep the reward even if the effort involved in creating the submission was marginally above the threshold. It therefore might be simpler to always hand out the reward in full, especially when the chance that exactly one contestant arrives at the contest is very low.

Fourth, in Section v of the online appendix we further discuss the possibility of allowing the designer to set some threshold t when there is only one participant. We show an example in which, when there are n contestants, a designer sets a threshold of $(1 - 1/n)R_i$ in equilibrium. Thus, neither setting $t = 0$ (allocating the item for free) nor setting $t = R_i$ (resulting in zero utility for the contestant) is an equilibrium. Notice that, in this case, setting $t = R_i$ corresponds to choosing the maximal rent dissipation contest, and therefore our conclusion that MRD contests form an equilibrium no longer holds. Nevertheless, as n increases the threshold $t = (1 - 1/n)R_i \to R_i$ approaches that of an MRD contest.

Previous literature, in particular Burdett et al. (2001), considers reservation prices which they term posted prices as it is in an auction context. However, their space of contest design allows only for a certain restricted type of posted price contests (in particular only posted prices that do not depend on the number of contestants) and does not allow for any other type of contest including for example general posted prices and/or Tullock contests. An interesting extension of our work can be to combine it with a generalization of the approach of Burdett et al. (2001) and consider a contest design space that allows both fully allocating contests as well as general posted prices. In the online appendix we observe that neither our results nor the results of Burdett et al. (2001) apply and hence the competition equilibria with such a design space remain an open question.

Thus, we view this issue as an important conclusion that stems from our paper. Our results should motivate future studies of competition among contests to carefully generalize the contest design space.

[15] For example, in KAGGLE (https://www.kaggle.com/general/14510), which uses contests to solve data science challenges, there are over 8 million users (Wikipedia, 2022a), but currently only about 20 active contests (according to Kaggle's open-source client).

Declaration of competing interest

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Appendix A. Missing proofs from Section 2

A.1. Proof of Claim 2.3

Claim 2.3. *If C_i has MDU and $p < p'$, then $\beta(C_i, p) > \beta(C_i, p')$.*

Proof. By definition,

$$\beta(C_i, p) = (1-p)^{n-1} R_i + \sum_{k=1}^{n-1} \binom{n-1}{k} p^k (1-p)^{n-1-k} \gamma_{C_i}(k+1)$$
$$= (1-p)^{n-1} \frac{R_i}{2} + (1-p)^{n-1} \frac{R_i}{2} + \sum_{k=1}^{n-1} \binom{n-1}{k} p^k (1-p)^{n-1-k} \gamma_{C_i}(k+1).$$

Let

$$\tilde{\gamma}(k+1) = \begin{cases} \frac{R_i}{2} & k = 0; \\ \gamma_{C_i}(k+1) & \text{otherwise.} \end{cases}$$

We can write $\beta(C_i, p) = (1-p)^{n-1} \frac{R_i}{2} + \mathbb{E}_{k\sim \text{Bin}(n-1,p)}[\tilde{\gamma}(k+1)]$. We recall that $\gamma_{C_i}(k)$, the utility of a contestant in a contest with k contestants, is at most $\frac{R_i}{k}$; in particular, $\gamma_{C_i}(2) \leq \frac{R_i}{2}$. Therefore, the sequence

$$\tilde{\gamma}(1) = \frac{R_i}{2} \geq \tilde{\gamma}(2) \geq \cdots \geq \tilde{\gamma}(n)$$

is decreasing under the assumption that C_i has monotonically decreasing utility. Due to the first order stochastic dominance of a binomial distribution with a higher p parameter over another binomial distribution with a lower p parameter (see e.g., Wolfstetter (1999)), we have

$$\mathbb{E}_{k\sim \text{Bin}(n-1,p)}[\tilde{\gamma}(k+1)] \geq \mathbb{E}_{k\sim \text{Bin}(n-1,p')}[\tilde{\gamma}(k+1)]$$

which implies

$$\beta(C_i, p) = (1-p)^{n-1} \frac{R_i}{2} + \mathbb{E}_{k\sim \text{Bin}(n-1,p')}[\tilde{\gamma}(k+1)]$$
$$> (1-p')^{n-1} \frac{R_i}{2} + \mathbb{E}_{k\sim \text{Bin}(n-1,p')}[\tilde{\gamma}(k+1)] = \beta(C_i, p'). \quad \square$$

Appendix B. Missing proofs from Section 3

Throughout this section we assume that $\mathcal{S}_i \subseteq C_{R_i}$ contains a maximal rent dissipation contest with monotonically decreasing utility, which we denote by $T_i \in \text{MRD}(\mathcal{S}_i)$.

B.1. Analysis of the contest competition game: proof of Theorem 3.1

Theorem 3.1 immediately follows from the following lemma, which is our main technical lemma. It shows that for any designer i, choosing T_i is always a best response if other designers choose contests with monotonically decreasing utility.

Lemma B.1. *Fix any $CCG(m, n, (R_i)_{i=1}^m, (\mathcal{S}_i)_{i=1}^m)$ where for any i, $\mathcal{S}_i \subseteq \mathcal{C}_{R_i}$ contains a maximal rent dissipation contest T_i that has monotonically decreasing utility. Fix some designer i and for all $j \neq i$ fix some $\hat{C}_j \in \mathcal{S}_j$ with monotonically decreasing utility. Then $u_i(T_i, \hat{\boldsymbol{C}}_{-i}) \geq u_i(C_i, \hat{\boldsymbol{C}}_{-i})$ for all $C_i \in \mathcal{S}_i$.*

We prove a useful claim before proving the lemma. The claim is about the change of contestants' participation equilibrium when a designer switches her contest. Intuitively, we expect that with all other contests left unchanged, a designer (say, designer 1) setting her contest to have less rent dissipation results in higher participation (properties (a) and (b) in the next claim). A more surprising property is that if participants are unwilling to participate in contest 1 under maximal rent dissipation, then no other contest will be lucrative enough to attract them (property (c)).

Claim B.1. *Following the notations in Lemma B.1, we let $(\hat{p}_1, \hat{p}_2, \dots, \hat{p}_m) = \boldsymbol{p}(T_1, \hat{C}_2, \dots, \hat{C}_m)$, and for any $C_1 \in \mathcal{S}_1$, let $(p_1, p_2, \dots, p_m) = \boldsymbol{p}(C_1, \hat{C}_2, \dots, \hat{C}_m)$. Then,*

(a) $p_1 \geq \hat{p}_1$;
(b) $p_j \leq \hat{p}_j$ *for all* $j \in \{2, \dots, m\}$;
(c) $\hat{p}_i = 0 \implies p_i = 0$ *for all* $i \in \{1, 2, \dots, m\}$.

Proof of (a). Assume by contradiction $\hat{p}_1 > p_1 \geq 0$, then there is some designer $j \neq 1$ with $\hat{p}_j < p_j$, because the probabilities sum to one. We now have the contradiction:

$$\begin{aligned}
&\beta(T_1, \hat{p}_1) \geq && (\hat{p}_1 > 0, \text{Claim 2.1})\\
&\beta(\hat{C}_j, \hat{p}_j) > && (\hat{p}_j < p_j, \hat{C}_j \text{has MDU, Claim 2.3})\\
&\beta(\hat{C}_j, p_j) \geq && (p_j > 0, \text{Claim 2.1})\\
&\beta(C_1, p_1) \geq && (T_1 \in \text{MRD}(\mathcal{S}_1), \text{Claim 2.4})\\
&\beta(T_1, p_1) > && (p_1 < \hat{p}_1, T_1 \text{has MDU, Claim 2.3})\\
&\beta(T_1, \hat{p}_1).
\end{aligned}$$

Proof of (b). From (a) we know

$$\sum_{j=2}^{m} \hat{p}_j = 1 - \hat{p}_1 \geq 1 - p_1 = \sum_{j=2}^{m} p_j. \tag{14}$$

Assume by contradiction that for some $j_0 \in \{2, 3, \dots, m\}$, $p_{j_0} > \hat{p}_{j_0} \geq 0$, then by (14) there must exist $j \in \{2, 3, \dots, m\}$ with $j \neq j_0$ such that $\hat{p}_j > p_j \geq 0$. We therefore have the contradiction

$$\begin{aligned}
&\beta(\hat{C}_j, p_j) > && (p_j < \hat{p}_j, \hat{C}_j \text{has MDU, Claim 2.3})\\
&\beta(\hat{C}_j, \hat{p}_j) \geq && (\hat{p}_j > 0, \text{Claim 2.1})\\
&\beta(\hat{C}_{j_0}, \hat{p}_{j_0}) > && (\hat{p}_{j_0} < p_{j_0}, \hat{C}_{j_0} \text{has MDU, Claim 2.3})\\
&\beta(\hat{C}_{j_0}, p_{j_0}) \geq && (p_{j_0} > 0, \text{Claim 2.1})\\
&\beta(\hat{C}_j, p_j).
\end{aligned}$$

Proof of (c). For any $i \in \{2, \dots, m\}$, (c) is an immediate corollary of (b). Consider $i = 1$, and assume by contradiction that $p_1 > \hat{p}_1 = 0$. Then there is some designer $j \neq 1$ with $p_j < \hat{p}_j$ because the probabilities sum to one. We then have the contradiction

$$\begin{aligned}
&\beta(T_1, \hat{p}_1) = && (\hat{p}_1 = 0)\\
&\gamma_{T_1}(1) = && (\text{By definition})\\
&R_1 \geq && \left(\gamma_{C_1}(k+1) \leq \frac{R_1}{k+1} \leq R_1 \text{by Eq. (1)}\right)\\
&\mathbb{E}_{k \sim \text{Bin}(n-1, p_1)}\left[\gamma_{C_1}(k+1)\right] = && \\
&\beta(C_1, p_1) \geq && (p_1 > 0, \text{Claim 2.1})\\
&\beta(\hat{C}_j, p_j) > && (p_j < \hat{p}_j, \hat{C}_j \text{has MDU, Claim 2.3})\\
&\beta(\hat{C}_j, \hat{p}_j) \geq && (\hat{p}_j > 0, \text{Claim 2.1})\\
&\beta(T_1, \hat{p}_1). \quad \square
\end{aligned}$$

Proof of Lemma B.1. Without loss of generality, we only prove it for designer $i = 1$. Suppose that when designer 1 chooses T_1 and all other designers j choose some contests $\hat{C}_j$ with monotonically decreasing utility, contestants choose participation probabilities $(\hat{p}_1, \hat{p}_2, \dots, \hat{p}_m) = \boldsymbol{p}(T_1, \hat{C}_2, \dots, \hat{C}_m)$. According to Claim 2.2, the expected utility of designer 1, denoted by $\hat{u}_1$, equals

$$\hat{u}_1 = u_1(T_1, \hat{C}_2, \dots, \hat{C}_m) = R_1 \left[1 - (1 - \hat{p}_1)^n (1 - \alpha_1)\right] - n\hat{p}_1 \beta(T_1, \hat{p}_1). \tag{15}$$

When designer 1 switches to any other contest $C_1 \in \mathcal{S}_1$, letting $(p_1, p_2, \dots, p_m) = \boldsymbol{p}(C_1, \hat{C}_2, \dots, \hat{C}_m)$, the expected utility of designer 1 becomes

$$u_1 = u_1(C_1, \hat{C}_2, \dots, \hat{C}_m) = R_1 \left[1 - (1 - p_1)^n (1 - \alpha_1)\right] - np_1 \beta(C_1, p_1). \tag{16}$$

Our goal is to show that $\hat{u}_1 \ge u_1$.

If $\hat{p}_1 = 0$, then by (c) of Claim B.1 we have $p_1 = 0$ and hence $\hat{u}_1 = u_1 = \alpha_1 R_1$. The conclusion holds.

Now assume $\hat{p}_1 > 0$. If $\hat{p}_j = 0$ for all $j \in \{2, \dots, m\}$, then by (c) of Claim B.1 we have $p_j = 0$ for all j. This implies $\hat{p}_1 = p_1 = 1$ and hence $\hat{u}_1 = R_1 - n\gamma_{T_1}(n)$ and $u_1 = R_1 - n\gamma_{C_1}(n)$. Since $\gamma_{T_1}(n) \le \gamma_{C_1}(n)$ by the assumption that T_1 has maximal rent dissipation, we have $\hat{u}_1 \ge u_1$.

Now we consider the case where $\hat{p}_j > 0$ for some $j \in \{2, \dots, m\}$. Because each contestant participates in both T_1 and $\hat{C}_j$ with positive probability, by equilibrium condition (Claim 2.1), we must have

$$\beta(T_1, \hat{p}_1) = \beta(\hat{C}_j, \hat{p}_j).$$

By (a) of Claim B.1, $p_1 \ge \hat{p}_1 > 0$, so each contestant participates in C_1 with positive probability, and by equilibrium condition (Claim 2.1),

$$\beta(C_1, p_1) \ge \beta(\hat{C}_j, p_j).$$

According to Claim 2.3, $\beta(\hat{C}_j, p)$ is a monotonically decreasing function of p, and by (b) of Claim B.1, $p_j \le \hat{p}_j$. Therefore, we have $\beta(\hat{C}_j, p_j) \ge \beta(\hat{C}_j, \hat{p}_j)$ and hence

$$\beta(C_1, p_1) \ge \beta(\hat{C}_j, p_j) \ge \beta(\hat{C}_j, \hat{p}_j) = \beta(T_1, \hat{p}_1). \tag{17}$$

Plugging (17) into (16), we get

$$u_1 \le R_1[1 - (1 - p_1)^n (1 - \alpha_1)] - np_1 \beta(T_1, \hat{p}_1).$$

Now we define function

$$f(p) = R_1[1 - (1 - p)^n (1 - \alpha_1)] - np\beta(T_1, \hat{p}_1). \tag{18}$$

We take its derivative:

$$\begin{aligned}
f'(p) &= nR_1(1 - p)^{n-1}(1 - \alpha_1) - n\beta(T_1, \hat{p}_1) \\
&= nR_1(1 - p)^{n-1}(1 - \alpha_1) - n\sum_{k=0}^{n-1} \binom{n-1}{k} \hat{p}_1^k (1 - \hat{p}_1)^{n-1-k} \gamma_{T_1}(k + 1) \\
&= nR_1(1 - p)^{n-1}\underbrace{(1 - \alpha_1)}_{\le 1} - n(1 - \hat{p}_1)^{n-1} R_1 - \underbrace{n\sum_{k=1}^{n-1} \binom{n-1}{k} \hat{p}_1^k (1 - \hat{p}_1)^{n-1-k} \gamma_{T_1}(k + 1)}_{\ge 0} \\
&\le nR_1(1 - p)^{n-1} - nR_1(1 - \hat{p}_1)^{n-1}.
\end{aligned}$$

For $p > \hat{p}_1$, we have $(1 - p)^{n-1} < (1 - \hat{p}_1)^{n-1}$, so $f'(p) < 0$. Thus, $f(p)$ is monotonically decreasing in the range $[\hat{p}_1, 1]$, which implies

$$\hat{u}_1 = f(\hat{p}_1) \ge f(p_1) \ge u_1, \tag{19}$$

concluding the proof.

B.2. Full characterization of equilibria for MDU contests: proof of Theorem 3.3

Additional properties of contestants' participation game. The following two claims use the notation and assumptions of Lemma B.1, specifically, we fix a contest competition game $CCG(m,n,(R_i)_{i=1}^m,(\mathcal{S}_i)_{i=1}^m)$, where for every i, MRD($\mathcal{S}_i$) contains at least one contest T_i, and T_i has monotonically decreasing utility. The first claim and its proof are similar to item (c) of Claim B.1.[16]

Claim B.2. *Let $\tilde{p}_i = p_i(T_1,\dots,T_m)$. For any strategy profile $(C_1,\dots,C_m) \in \mathcal{S}_1 \times \cdots \times \mathcal{S}_m$, let $p_i = p_i(C_1,\dots,C_m)$. Then for any $i \in \{1,\dots,m\}$, $\tilde{p}_i = 0$ implies $p_i = 0$.*

Proof. Assume by contradiction there exists i such that $\tilde{p}_i = 0$ and $p_i > 0$. Then there exists $j \neq i$ such that $\tilde{p}_j > p_j$. By Claim 2.1 we have $\beta(T_j,\tilde{p}_j) \geq \beta(T_i,\tilde{p}_i) = R_i$ and $\beta(C_i,p_i) \geq \beta(C_j,p_j)$. By Claim 2.4 (since $T_j \in$ MRD($\mathcal{S}_j$)), we have $\beta(C_j,p_j) \geq \beta(T_j,p_j)$. By Claim 2.3, since T_j is a MDU contest and $\tilde{p}_j > p_j$, we have $\beta(T_j,p_j) > \beta(T_j,\tilde{p}_j)$. Finally, it is obvious that $\beta(C_i,p_i) \leq R_i$. Combining these inequalities, we get

$$\beta(T_j,\tilde{p}_j) \geq \beta(T_i,\tilde{p}_i) = R_i \geq \beta(C_i,p_i) \geq \beta(C_j,p_j) \geq \beta(T_j,p_j) > \beta(T_j,\tilde{p}_j),$$

which is a contradiction. □

When $\hat{\boldsymbol{C}}_{-i}$ are all MDU contests, Lemma B.1 states that T_i is a dominant contest for designer i. Thus, if we have $u_i(T_i,\hat{\boldsymbol{C}}_{-i}) = u_i(C_i,\hat{\boldsymbol{C}}_{-i})$ for some $C_i \in \mathcal{S}_i$, then C_i is a best response to $\hat{\boldsymbol{C}}_{-i}$. The next claim shows that, in this case, the equilibrium outcome in the two contestants' participation games $(T_i,\hat{\boldsymbol{C}}_{-i})$ and $(C_i,\hat{\boldsymbol{C}}_{-i})$ is identical.

Claim B.3. *If $u_i(T_i,\hat{\boldsymbol{C}}_{-i}) = u_i(C_i,\hat{\boldsymbol{C}}_{-i})$ for some $C_i \in \mathcal{S}_i$, then $\hat{p}_i = p_i$ and $\beta(T_i,\hat{p}_i) = \beta(C_i,p_i)$, where $\hat{p}_i = p_i(T_i,\hat{\boldsymbol{C}}_{-i})$ and $p_i = p_i(C_i,\hat{\boldsymbol{C}}_{-i})$.*

Proof. Without loss of generality, we only prove it for contest designer $i = 1$. Following the notation in Section B.1 (Eq. (15), Eq. (16) and Eq. (18)), define

$$\hat{u}_1 = u_1(T_1,\hat{\boldsymbol{C}}_{-1}) = R_1\left[1-(1-\alpha_1)(1-\hat{p}_1)^n\right] - n\hat{p}_1\beta(T_1,\hat{p}_1),$$
$$u_1 = u_1(C_1,\hat{\boldsymbol{C}}_{-1}) = R_1\left[1-(1-\alpha_1)(1-p_1)^n\right] - np_1\beta(C_1,p_1),$$
$$f(p) = R_1[1-(1-\alpha_1)(1-p)^n] - np\beta(T_1,\hat{p}_1).$$

Recall that by the assumption in the statement of the claim we have $\hat{u}_1 = u_1$. Consider the following three cases:

- If $\hat{p}_1 = 0$, then by (c) of Claim B.1, $p_1 = 0 = \hat{p}_1$. And $\beta(T_1,\hat{p}_1) = \beta(C_1,p_1) = R_1$.
- If $\hat{p}_1 = 1$, then by (a) of Claim B.1, $1 \geq p_1 \geq \hat{p}_1 = 1$ hence $p_1 = \hat{p}_1 = 1$. Therefore,

$$R_1 - n\beta(T_1,\hat{p}_1) = \hat{u}_1 = u_1 = R_1 - n\beta(C_1,p_1).$$

  This immediately implies $\beta(T_1,\hat{p}_1) = \beta(C_1,p_1)$.
- Otherwise, $0 < \hat{p}_1 < 1$, so there exists some $j \in \{2,\dots,m\}$ such that $\hat{p}_j > 0$ and therefore Eq. (19) in the proof of Lemma B.1 holds. Furthermore, since $\hat{u}_1 = u_1$, all the inequalities in (19) become equalities. Thus,

$$f(\hat{p}_1) = f(p_1) = u_1. \tag{20}$$

  By (a) of Claim B.1, $p_1 \geq \hat{p}_1$, so by strict monotonicity of $f(p)$ in the range $p \in [\hat{p}_1,1]$, Eq. (20) implies $p_1 = \hat{p}_1$ and in addition

$$R_1[1-(1-\alpha_1)(1-p_1)^n] - np_1\beta(T_1,\hat{p}_1) = f(p_1)$$
$$= u_1 = R_1[1-(1-\alpha_1)(1-p_1)^n] - np_1\beta(C_1,p_1),$$

  which directly implies $\beta(T_1,\hat{p}_1) = \beta(C_1,p_1)$ since $p_1 = \hat{p}_1 > 0$. □

Proof of the "$\Longrightarrow$" direction of items 2 and 3 of Theorem 3.3. Under the notation of Theorem 3.3, for every contestant-symmetric subgame-perfect equilibrium $(C_1,\dots,C_m)$, recall that Supp$(C_1,\dots,C_m) = \{i : p_i(C_1,\dots,C_m) > 0\}$. We defer the proof that Supp$(C_1,\dots,C_m) = P$ and first prove a useful lemma:

Lemma B.2.

- *If $|\text{Supp}(C_1,\dots,C_m)| > 1$ then for any $i \in \text{Supp}(C_1,\dots,C_m)$, $C_i \in \text{MRD}(\mathcal{S}_i)$.*

[16] We note the differences: (1) here every contest changes, while in Claim B.1 only one contest changes, and (2) here we require all contests to be maximal rent dissipation (and MDU), while in Claim B.1 we only require them to be MDU.

- *If* $\mathrm{Supp}(C_1,\dots,C_m)=\{i_0\}$ *then* $p_{i_0}(C_1,\dots,C_m)=1$ *and* $\gamma_{C_{i_0}}(n)=\gamma_{T_{i_0}}(n)$.

Proof. Recall that T_i is a dominant contest and hence a best response to $\boldsymbol{C}_{-i}$. Since $(C_1,\dots,C_m)$ is a contestant-symmetric subgame-perfect equilibrium, C_i is also a best response. Applying Claim B.3, we get $p_i := p_i(C_i,\boldsymbol{C}_{-i}) = p_i(T_i,\boldsymbol{C}_{-i})$ and

$$\beta(T_i,p_i)=\beta(T_i,p_i(T_i,\boldsymbol{C}_{-i}))=\beta(C_i,p_i(C_i,\boldsymbol{C}_{-i}))=\beta(C_i,p_i). \tag{21}$$

By definition

$$\beta(T_i,p_i)=\sum_{k=0}^{n-1}\binom{n-1}{k}p_i^k(1-p_i)^{n-1-k}\gamma_{T_i}(k+1)$$
$$=\beta(C_i,p_i)=\sum_{k=0}^{n-1}\binom{n-1}{k}p_i^k(1-p_i)^{n-1-k}\gamma_{C_i}(k+1).$$

If $p_i=1$, then $\beta(T_i,p_i)=\gamma_{T_i}(n)=\beta(C_i,p_i)=\gamma_{C_i}(n)$, this corresponds to the second case of the lemma.

Otherwise, for any $i\in\{1,\dots,m\}$, $p_i<1$. We prove the first case of the lemma, i.e. for any $i\in\mathrm{Supp}(C_1,\dots,C_m)$, $C_i\in\mathrm{MRD}(\mathcal{S}_i)$. Actually, as $0<p_i<1$, $\binom{n-1}{k}p_i^k(1-p_i)^{n-1-k}>0$ for every $k=0,\dots,n-1$. Moreover, $T_i\in\mathrm{MRD}(\mathcal{S}_i)$ implies $\gamma_{T_i}(k+1)\le\gamma_{C_i}(k+1)$ for every $k=0,\dots,n-1$. As a result, for (21) to hold, we must have $\gamma_{T_i}(k+1)=\gamma_{C_i}(k+1)$ for every $k=0,\dots,n-1$, which indicates $C_i\in\mathrm{MRD}(\mathcal{S}_i)$. This completes the proof of the lemma. □

Comparing Lemma B.2 with the conclusion of the " $\Longrightarrow$ " direction, we are left to prove $\mathrm{Supp}(C_1,\dots,C_m)=P$. The $\mathrm{Supp}(C_1,\dots,C_m)\subseteq P$ result is just a direct implication of Claim B.2. We then prove $P\subseteq\mathrm{Supp}(C_1,\dots,C_m)$. Denote $p_i(C_1,\dots,C_m)$ by p_i for simplicity. Assume towards a contradiction that there exists $i\in\{1,\dots,m\}$ such that $\tilde p_i>p_i=0$. Then there exists $j\ne i$ such that $p_j>\tilde p_j$. Note that this implies $p_j>0$. Therefore, by Lemma B.2, either $C_j\in\mathrm{MRD}(\mathcal{S}_j)$, or $p_j=1$ and $\gamma_{C_j}(n)=\gamma_{T_j}(n)$. In either case we have $\beta(T_j,p_j)=\beta(C_j,p_j)$. By equilibrium condition (Claim 2.1), $\beta(C_j,p_j)\overset{(p_j>0)}{\ge}\beta(C_i,p_i)\overset{(p_i=0)}{=}R_i$ and $\beta(T_i,\tilde p_i)\overset{(\tilde p_i>0)}{\ge}\beta(T_j,\tilde p_j)$. By Claim 2.3, $\beta(T_j,\tilde p_j)>\beta(T_j,p_j)$ and $R_i=\beta(T_i,p_i)>\beta(T_i,\tilde p_i)$. These inequalities together yield

$$\beta(T_j,\tilde p_j)>\beta(T_j,p_j)=\beta(C_j,p_j)\ge\beta(C_i,p_i)=R_i=\beta(T_i,p_i)>\beta(T_i,\tilde p_i)\ge\beta(T_j,\tilde p_j),$$

which is a contradiction. Therefore, we conclude that $p_i=0$ implies $\tilde p_i=0$ for any $i\in\{1,\dots,m\}$. This completes the proof.

Proof of the "⟸" direction of items 2 and 3 of Theorem 3.3. To prove this direction we first assume $|P|>1$. Assume $(C_1,\dots,C_m)$ is any strategy profile satisfying $C_i\in\mathrm{MRD}(\mathcal{S}_i)$ for any $i\in P$. To prove that it is a contestant-symmetric subgame-perfect equilibrium, we only need to show that for any i, C_i is designer i's best response when the other designers choose $\boldsymbol{C}_{-i}$. Note that Lemma B.1 already guarantees that for $i\in P$, C_i is designer i's best response, so we are left to show that this also holds for those $i\notin P$. Assume $\tilde p_i=0$, then for any $C_i'\in\mathcal{S}_i$, by Claim B.2, $p_i(C_i',\boldsymbol{C}_{-i})=0$, which implies that designer i gets zero utility no matter which C_i' she chooses. So C_i is indeed one of her best responses. To conclude, C_i is designer i's best response for any i, which implies that $(C_1,\dots,C_m)$ is a contestant-symmetric subgame-perfect equilibrium.

We then assume $|P|=1$. Suppose $P=\{i_0\}$, and assume $(C_1,\dots,C_m)$ is any strategy profile satisfying $\gamma_{C_{i_0}}(n)=\gamma_{T_{i_0}}(n)$. We need to show that for any i, C_i is designer i's best response when the other designers choose $\boldsymbol{C}_{-i}$. This time Claim B.2 promises that for any $i\ne i_0$ and any $C_i'\in\mathcal{S}_i$, $p_i(C_i',\boldsymbol{C}_{-i})=0$, so C_i is designer i's best response. And by the same claim, $p_i(C_{i_0},\boldsymbol{C}_{-i_0})=p_i(T_{i_0},\boldsymbol{C}_{-i_0})=0$ for any $i\ne i_0$, so $p_{i_0}(C_{i_0},\boldsymbol{C}_{-i_0})=p_{i_0}(T_{i_0},\boldsymbol{C}_{-i_0})=1$. As a result,

$$\beta(C_{i_0},p_{i_0}(C_{i_0},\boldsymbol{C}_{-i_0}))=\gamma_{C_{i_0}}(n)=\gamma_{T_{i_0}}(n)=\beta(T_{i_0},p_{i_0}(T_{i_0},\boldsymbol{C}_{-i_0})),$$

and

$$u_{i_0}(C_{i_0},\boldsymbol{C}_{-i_0})=R_{i_0}-n\beta(C_{i_0},p_{i_0}(C_{i_0},\boldsymbol{C}_{-i_0}))=R_{i_0}-n\beta(T_{i_0},p_{i_0}(T_{i_0},\boldsymbol{C}_{-i_0}))=u_{i_0}(T_{i_0},\boldsymbol{C}_{-i_0}).$$

In other words, C_{i_0} has equal utility for designer i_0 as her best response T_{i_0}, which implies that C_{i_0} is also designer i_0's best response. To conclude, C_i is designer i's best response for any i, so $(C_1,\dots,C_m)$ is a contestant-symmetric subgame-perfect equilibrium. This completes the proof.

Proof of item 1 of Theorem 3.3. For any contestant-symmetric subgame-perfect equilibrium $(C_1,\dots,C_m)$, we claim that $(\tilde p_1,\dots,\tilde p_m)$ is a symmetric equilibrium for the contestants under $(C_1,\dots,C_m)$; then, since the contestants' symmetric equilibrium is unique according to Lemma 2.8, we must have $p_i(C_1,\dots,C_m)=\tilde p_i$, which completes the proof. Consider $\beta(C_i,\tilde p_i)$ for all $i\in\{1,\dots,m\}$. If $i\notin P$, then $\tilde p_i=0$ and $\beta(C_i,\tilde p_i)=R_i=\beta(T_i,\tilde p_i)$. If $i\in P$, then by item 2 and 3, either $C_i\in\mathrm{MRD}(\mathcal{S}_i)$ or $\tilde p_i=1$ and $\gamma_{C_i}(n)=\gamma_{T_i}(n)$, and we have $\beta(C_i,\tilde p_i)=\beta(T_i,\tilde p_i)$ in either case. So $\beta(C_i,\tilde p_i)=\beta(T_i,\tilde p_i)$ for all $i\in\{1,\dots,m\}$. Then applying equilibrium condition (Claim 2.1) for the case where designers choose $(T_1,\dots,T_m)$, we get $\beta(C_i,\tilde p_i)=\beta(T_i,\tilde p_i)=\beta(T_j,\tilde p_j)=\beta(C_j,\tilde p_j)\ge\beta(T_\ell,\tilde p_\ell)=\beta(C_\ell,\tilde p_\ell)$ for any $i,j\in P$ and $\ell\notin P$. We therefore know that when designers choose $(C_1,\dots,C_m)$, $(\tilde p_1,\dots,\tilde p_m)$ is still a best response for any contestant when all the other contestants use $(\tilde p_1,\dots,\tilde p_m)$, which means that $(\tilde p_1,\dots,\tilde p_m)$ is a contestants' symmetric equilibrium.

B.3. Pareto efficiency of the equilibria: proof of Theorem 3.7

Assume $\boldsymbol{T} = (T_1, \ldots, T_m)$ is the contestant-symmetric subgame-perfect equilibrium in Theorem 3.1 for $CCG(m, n, (R_i)_{i=1}^n, (S_i)_{i=1}^n)$, and $\boldsymbol{C} = (C_1, \ldots, C_m) \in \mathcal{S}_1 \times \cdots \times \mathcal{S}_m$ is any other strategy profile of the designers. We will show that $\boldsymbol{C}$ is not a Pareto improvement of $\boldsymbol{T}$ which proves the theorem. Denote by $(\tilde{p}_1, \ldots, \tilde{p}_m) = \boldsymbol{p}(\boldsymbol{T})$ and $(p_1, \ldots, p_m) = \boldsymbol{p}(\boldsymbol{C})$ the symmetric equilibria contestants play under $\boldsymbol{T}$ and $\boldsymbol{C}$, respectively. If $p_i = \tilde{p}_i$ for any i, then as T_i is the maximal rent dissipation contest of $\mathcal{S}_i$, by Claim 2.4 we have

$$\beta(T_i, p_i) \le \beta(C_i, p_i)$$

As a result, for any designer i,

$$\begin{aligned} u_i(\boldsymbol{T}) &= R_i[1 - (1 - \alpha_i)(1 - \tilde{p}_i)^n] - n\tilde{p}_i\beta(T_i, \tilde{p}_i) \\ &= R_i[1 - (1 - \alpha_i)(1 - p_i)^n] - np_i\beta(T_i, p_i) \\ &\ge R_i[1 - (1 - \alpha_i)(1 - p_i)^n] - np_i\beta(C_i, p_i) = u_i(\boldsymbol{C}), \end{aligned}$$

showing that $\boldsymbol{C}$ is not a Pareto improvement of $\boldsymbol{T}$.

Otherwise, there exist i, j such that $p_i > \tilde{p}_i$ and $p_j < \tilde{p}_j$. Note that this implies $p_i, \tilde{p}_j > 0$, so by equilibrium condition (Claim 2.1), we get

$$\beta(C_i, p_i) \ge \beta(C_j, p_j), \tag{22}$$

and

$$\beta(T_j, \tilde{p}_j) \ge \beta(T_i, \tilde{p}_i). \tag{23}$$

Since $p_j < \tilde{p}_j$ and T_j is a monotonically decreasing utility contest, by Claim 2.3 we have

$$\beta(T_j, \tilde{p}_j) < \beta(T_j, p_j). \tag{24}$$

Moreover, as T_j is a maximal rent dissipation contest in $\mathcal{S}_j$, we have

$$\beta(T_j, p_j) \le \beta(C_j, p_j) \tag{25}$$

by Claim 2.4. Combining these inequalities together, we get

$$\beta(T_i, \tilde{p}_i) \overset{(23)}{\le} \beta(T_j, \tilde{p}_j) \overset{(24)}{<} \beta(T_j, p_j) \overset{(25)}{\le} \beta(C_j, p_j) \overset{(22)}{\le} \beta(C_i, p_i).$$

Now we consider the utilities of designer i in $\boldsymbol{T}$ and $\boldsymbol{C}$. We have

$$\begin{aligned} u_i(\boldsymbol{T}) &= R_i[1 - (1 - \alpha_i)(1 - \tilde{p}_i)^n] - n\tilde{p}_i\beta(T_i, \tilde{p}_i), \\ u_i(\boldsymbol{C}) &= R_i[1 - (1 - \alpha_i)(1 - p_i)^n] - np_i\beta(C_i, p_i). \end{aligned}$$

Similarly to (18), we define $f(p) = R_i[1 - (1 - \alpha_i)(1 - p)^n] - np\beta(T_i, \tilde{p}_i)$ and have

$$f'(p) \le (1 - \alpha_i)nR_i(1 - p)^{n-1} - nR_i(1 - \tilde{p}_i)^{n-1} < 0$$

for $p > \tilde{p}_i$, which implies that $f(p)$ is a strictly decreasing function of p when $p \ge \tilde{p}_i$. Therefore, as $p_i > \tilde{p}_i$, we have $f(p_i) < f(\tilde{p}_i)$. As a result,

$$\begin{aligned} u_i(\boldsymbol{T}) = f(\tilde{p}_i) > f(p_i) &= R_i[1 - (1 - \alpha_i)(1 - p_i)^n] - np_i\beta(T_i, \tilde{p}_i) \\ &\ge R_i[1 - (1 - \alpha_i)(1 - p_i)^n] - np_i\beta(C_i, p_i) = u_i(\boldsymbol{C}), \end{aligned}$$

which indicates that $\boldsymbol{C}$ cannot be a Pareto improvement of $\boldsymbol{T}$, concluding the proof.

B.4. Proof of Example 3.5

Example 3.5. Consider n contestants and $m \ge 3$ contest designers with $0 \le \alpha_i < 1$. Contest 1 has reward $R_1 = 1$, and each other contest has reward $R_j = \left(\frac{m-1}{m-2}\right)^{n-1} + 1$. Each set $\mathcal{S}_i$ contains all MDU contests (hence contains APA). Then for any contest $C_1 \in \mathcal{S}_1$, $(C_1, T_2, \ldots, T_m)$ where $T_j = \text{APA} \in \text{MRD}(\mathcal{S}_j)$ for $j = 2, \ldots, m$ is a contestant-symmetric SPE. In this equilibrium, $p_1(C_1, T_2, \ldots, T_m) = 0$, and $p_j(C_1, T_2, \ldots, T_m) = \frac{1}{m-1} > 0$ for any $j = 2, \ldots, m$.

Proof. When for all $i = 1, \ldots, m$, contest designer i chooses $T_i = \text{APA}$, we have that for any $j = 2, \ldots, m$, the contestant's utility satisfies

$$\beta\left(T_j, \frac{1}{m-1}\right) = R_j\left(1 - \frac{1}{m-1}\right)^{n-1} = \left[\left(\frac{m-1}{m-2}\right)^{n-1} + 1\right]\left(\frac{m-2}{m-1}\right)^{n-1}$$
$$> 1 = R_1 = \beta(T_1, 0),$$

which implies that $\left(0, \frac{1}{m-1}, \dots, \frac{1}{m-1}\right)$ is a symmetric equilibrium for contestants in $(T_1, \dots, T_m)$, i.e. $p_i(T_1, \dots, T_m) = \begin{cases} \frac{1}{m-1}, & i = 2, \dots, m \\ 0, & i = 1 \end{cases}$. Then the claim that any $(C_1, T_2, \dots, T_m)$, with T_j = APA for $j = 2, \dots, m$ and C_1 be an arbitrary contest in $\mathcal{S}_1$, is a contestant-symmetric subgame-perfect equilibrium with $p_i(C_1, T_2, \dots, T_m) = p_i(T_1, \dots, T_m) = \begin{cases} \frac{1}{m-1}, & i = 2, \dots, m \\ 0, & i = 1 \end{cases}$ follows from Theorem 3.3. □

Appendix C. Missing proofs from Section 4

C.1. Proof of Lemma 4.1

Lemma 4.1 *(Corollary of Baye et al. (1996); Schweinzer and Segev (2012); Ewerhart (2017)). A contestant's utility in a Tullock contest C_τ with parameter $\tau \in [0, +\infty]$, reward R, and k total contestants equals*

$$\gamma_{C_\tau}(k) = \begin{cases} R & \text{if } k = 1, \\ R(\frac{1}{k} - \frac{k-1}{k^2}\tau) & \text{if } \frac{k}{k-1} \geq \tau, \\ 0 & \text{if } \frac{k}{k-1} < \tau \leq +\infty. \end{cases} \tag{12}$$

As corollaries,

- *Every Tullock contest satisfies MDU.*
- *Any Tullock contest with $\tau \geq 2$ has full rent dissipation.*
- *If $\mathcal{S}$ is the set of all Tullock contests with parameter τ in some range whose maximum $\tau^{\max}$ is well defined and at most 2, then the Tullock contest with $\tau^{\max}$ is the only contest in* MRD($\mathcal{S}$).

Proof. The expression for $\gamma_{C_\tau}(k)$ follows from previous works. Specifically, when $k = 1$ the single contestant exerts no effort and obtains utility $\gamma_{C_\tau}(1) = R$. For $k \geq 2$, Proposition 2 of Schweinzer and Segev (2012) shows that if $\frac{k}{k-1} \geq \tau$ then $\gamma_{C_\tau}(k) = R(\frac{1}{k} - \frac{k-1}{k^2}\tau)$. Corollary 5 of Ewerhart (2017) shows that if $\frac{k}{k-1} < \tau < +\infty$ then $\gamma_{C_\tau}(k) = 0$. Baye et al. (1996) show that for $\tau = +\infty$, $\gamma_{C_\tau}(k) = 0$ for $k \geq 2$.

Now, we prove the first corollary that every Tullock contest satisfies MDU. To prove this, we only need to verify that the term $\gamma_{C_\tau}(k) = R(\frac{1}{k} - \frac{k-1}{k^2}\tau)$ is monotonically decreasing in k (when $2 \leq \frac{k}{k-1} < \tau$). Consider the function $f(x) = \frac{1}{x} - \frac{x-1}{x^2}\tau$ for x satisfying $2 \leq \frac{x}{x-1} < \tau$. We take its derivative:

$$f'(x) = -\frac{1}{x^2} - \frac{x^2 - 2(x-1)x}{x^4}\tau = -\frac{1}{x^2} + \frac{x-2}{x^3}\tau.$$

Given $\frac{x}{x-1} < \tau$, we have

$$f'(x) \leq -\frac{1}{x^2} + \frac{x-2}{x^3}\frac{x}{x-1} = \frac{1}{x^2}\left(-1 + \frac{x-2}{x-1}\right) = -\frac{1}{x^2} \cdot \frac{1}{x-1} \leq 0.$$

Thus, $f(x)$ is monotonically decreasing, and so is $\gamma_{C_\tau}(k)$.

Then, we prove the second corollary. With $\tau > 2$, we have $\frac{k}{k-1} < \tau$ for any $k \geq 2$, so $\gamma_{C_\tau}(k) = 0$, meaning that C_τ has full rent dissipation. With $\tau = 2$, we have $\gamma_{C_\tau}(k = 2) = R(\frac{1}{k} - \frac{k-1}{k^2}\tau) = 0$ and $\gamma_{C_\tau}(k) = 0$ for any $k \geq 3$ since $\frac{k}{k-1} < \tau$, so C_τ has full rent dissipation.

Finally, we prove the third corollary. We note that, for any fixed $k \geq 2$, the contestant's utility $\gamma_{C_\tau}(k) = R(\frac{1}{k} - \frac{k-1}{k^2}\tau)$ is a strictly decreasing non-negative function of τ when $0 \leq \tau \leq \frac{k}{k-1} \leq 2$ and $\gamma_{C_\tau}(k) = 0$ when $\tau > \frac{k}{k-1}$. This means that the contest with the largest parameter $\tau^{\max} \leq 2$ is the only MRD contest among Tullock contests with parameters ≤ 2. □

Appendix D. Missing parts from Section 5

D.1. Proof of Theorem 5.1

For any $\boldsymbol{C} = (C_1, \dots, C_m)$, let $\boldsymbol{p}(\boldsymbol{C}) = (p_1, \dots, p_m)$. By Eq. (13) and the assumption that $\alpha_i = \alpha$,

$$W_{\mathcal{S}}(C_1, \dots, C_m) = \sum_{i=1}^{m} R_i - (1-\alpha)\sum_{i=1}^{m} R_i(1-p_i)^n.$$

Then by Hölder's inequality,

$$\left(\sum_{i=1}^{m} R_i(1-p_i)^n\right)^{\frac{1}{n}} \left(\sum_{i=1}^{m} R_i^{-\frac{1}{n-1}}\right)^{\frac{n-1}{n}} = \left(\sum_{i=1}^{m}\left(R_i^{\frac{1}{n}}(1-p_i)\right)^n\right)^{\frac{1}{n}} \left(\sum_{i=1}^{m}\left(R_i^{-\frac{1}{n}}\right)^{\frac{n}{n-1}}\right)^{\frac{n-1}{n}}$$

$$\geq \sum_{i=1}^{m}\left(R_i^{\frac{1}{n}}(1-p_i)\right)\left(R_i^{-\frac{1}{n}}\right) = \sum_{i=1}^{m}(1-p_i) = m-1.$$

So

$$\sum_{i=1}^{m} R_i(1-p_i)^n \geq \left(\frac{m-1}{\left(\sum_{i=1}^{m} R_i^{-\frac{1}{n-1}}\right)^{\frac{n-1}{n}}}\right)^n = \frac{(m-1)^n}{\left(\sum_{i=1}^{m} R_i^{-\frac{1}{n-1}}\right)^{n-1}},$$

and

$$W_S(C_1,\dots,C_m) = \sum_{i=1}^{m} R_i - (1-\alpha)\sum_{i=1}^{m} R_i(1-p_i)^n \leq \sum_{i=1}^{m} R_i - \frac{(1-\alpha)(m-1)^n}{\left(\sum_{i=1}^{m} R_i^{-\frac{1}{n-1}}\right)^{n-1}}. \tag{26}$$

We will show that $W_S(T_1,\dots,T_m)$ is equal to the right-hand side in the following cases.

The case when $\mathcal{S}_i$ contains a full rent dissipation contest for every i. In this case, since $T_i \in \mathrm{MRD}(\mathcal{S}_i)$, we have that T_i is also a full rent dissipation contest. Let $\tilde{p}_i = p_i(T_1,\dots,T_m)$. If $\tilde{p}_i = 0$ for some i, then by Claim B.2, $p_i(C_1,\dots,C_m) = 0$. Thus $W_S(\boldsymbol{T}) = W_S(\boldsymbol{T}_{-i})$ and $W_S(\boldsymbol{C}) = W_S(\boldsymbol{C}_{-i})$. We can thus assume that $\tilde{p}_i > 0$ for every i. Then by Claim 2.1,

$$\beta(T_1,\tilde{p}_1) = \beta(T_2,\tilde{p}_2) = \cdots = \beta(T_m,\tilde{p}_m). \tag{27}$$

As T_i has full rent dissipation, we have $\beta(T_i,\tilde{p}_i) = R_i(1-\tilde{p}_i)^{n-1}$. Substitute this into (27), we get

$$R_1^{\frac{1}{n-1}}(1-\tilde{p}_1) = R_2^{\frac{1}{n-1}}(1-\tilde{p}_2) = \cdots = R_m^{\frac{1}{n-1}}(1-\tilde{p}_m).$$

So $\forall i = 1,\dots,m$,

$$\frac{1-\tilde{p}_j}{1-\tilde{p}_i} = \frac{R_j^{-\frac{1}{n-1}}}{R_i^{-\frac{1}{n-1}}}, \quad \forall j = 1,\dots,m. \tag{28}$$

Note that $\sum_{j=1}^{m}(1-\tilde{p}_j) = m-1$. Fixing any i and summing (28) for $j=1$ to m, we obtain

$$\frac{m-1}{1-\tilde{p}_i} = \sum_{j=1}^{m}\frac{1-\tilde{p}_j}{1-\tilde{p}_i} = \frac{\sum_{j=1}^{m} R_j^{-\frac{1}{n-1}}}{R_i^{-\frac{1}{n-1}}}.$$

So

$$1-\tilde{p}_i = \frac{m-1}{\sum_{j=1}^{m} R_j^{-\frac{1}{n-1}}} \cdot R_i^{-\frac{1}{n-1}},$$

and therefore,

$$W_S(T_1,\dots,T_m) = \sum_{i=1}^{m} R_i - (1-\alpha)\sum_{i=1}^{m} R_i(1-\tilde{p}_i)^n = \sum_{i=1}^{m} R_i - \frac{(1-\alpha)(m-1)^n}{\left(\sum_{j=1}^{m} R_j^{-\frac{1}{n-1}}\right)^{n-1}}, \tag{29}$$

which meets the bound given by (26). This completes the proof for this case.

The case of MRD-symmetric strategy space. Note that by definition any two contests $T, T' \in \mathrm{MRD}(\mathcal{S}_i)$ must have the same $\boldsymbol{\gamma}$ vector (i.e., for any $k = 1,\dots,n$, $\gamma_T(k) = \gamma_{T'}(k)$). The following lemma therefore proves this case:

Lemma D.1. *In a $CCG(m, n, (R_i)_{i=1}^m, (\mathcal{S}_i)_{i=1}^m)$ with $R_1 = \cdots = R_m = R$ (and $\mathcal{S}_i$'s can be different), for any strategy profile $\boldsymbol{C}' = (C_1', \ldots, C_m')$ where for every i, $C_i' \in \mathcal{S}_i$ is a MDU contest, and has the same $\boldsymbol{\gamma}$ vector (i.e., for any $k = 1, \ldots, n$, and any $i, j \in \{1, \ldots, m\}$, $\gamma_{C_i'}(k) = \gamma_{C_j'}(k)$), $\boldsymbol{C}'$ maximizes W_S.*

Proof. When the rewards are the same, (26) becomes

$$W_S(C_1, \ldots, C_m) = mR - (1-\alpha)R\sum_{i=1}^m (1-p_i)^n \le mR - (1-\alpha)R\frac{(m-1)^n}{m^{n-1}}$$

for any $(C_1, \ldots, C_m) \in \mathcal{S}_1 \times \cdots \times \mathcal{S}_m$. On the other hand, since any two C_i' and C_j' have the same $\boldsymbol{\gamma}$ vector it follows that

$$\beta\left(C_1', \frac{1}{m}\right) = \cdots = \beta\left(C_m', \frac{1}{m}\right).$$

So $(p_1, \ldots, p_m) = \left(\frac{1}{m}, \ldots, \frac{1}{m}\right)$ is a symmetric equilibrium for the contestants under $(C_1', \ldots, C_m')$. Moreover, by Lemma 2.8, it is the unique symmetric equilibrium, so $p_i(\boldsymbol{C}') = \frac{1}{m}$. Therefore

$$\begin{aligned} W_S(C_1', \ldots, C_m') &= \sum_{i=1}^m R_i - (1-\alpha)\sum_{i=1}^m R_i(1-p_i(\boldsymbol{C}'))^n \\ &= mR - (1-\alpha)R\frac{(m-1)^n}{m^{n-1}} \ge W_S(C_1, \ldots, C_m), \end{aligned}$$

which completes our proof. ☐

D.2. Counter-examples to Theorem 5.1

Example D.2. Let $m = 2, n = 2, R_1 = R_2 = 1, \alpha_1 = \alpha_2 = 0$. Let C, T be Tullock contests with $\tau_C = 1, \tau_T = 1.2$. It can be verified that $\gamma_C = (1, \frac{1}{4})$ and $\gamma_T = (1, \frac{1}{5})$. Suppose $\mathcal{S}_1 = \{C\}$ and $\mathcal{S}_2 = \{C, T\}$. Then $\mathrm{MRD}(\mathcal{S}_1) = \{C\}$ and $\mathrm{MRD}(\mathcal{S}_2) = \{T\}$. Due to symmetry, $\boldsymbol{p}(C, C) = (\frac{1}{2}, \frac{1}{2})$, so

$$W_S(C, C) = 2 - 2\left(1 - \tfrac{1}{2}\right)^2 = \tfrac{3}{2}.$$

We can determine $\boldsymbol{p}(C, T) = (\tilde{p}_1, 1 - \tilde{p}_1)$ by solving $\beta(C, \tilde{p}_1) = 1 - \tilde{p}_1 + \frac{1}{4}\tilde{p}_1 = \beta(T, 1 - \tilde{p}_1) = \tilde{p}_1 + \frac{1}{5}(1 - \tilde{p}_1)$ hence $\tilde{p}_1 = \frac{16}{31}$ and $\tilde{p}_2 = 1 - \tilde{p}_1 = \frac{15}{31}$. So

$$W_S(C, T) = 2 - (1 - \tilde{p}_1)^2 - (1 - \tilde{p}_2)^2 = \tfrac{1441}{961} < \tfrac{3}{2} = W_S(C, C).$$

Remark D.3. One can construct a similar example for every m, n, and $R_1, \ldots, R_m$. Specifically, construct $\mathcal{S}_i$ as follows: consider equation set

$$R_i \sum_{k=1}^n \binom{n-1}{k-1} \tilde{p}_i^{k-1}(1 - \tilde{p}_i)^{n-k}\gamma_{C_i}(k) = R, \quad \forall i = 1, \ldots, m.$$

Find a solution $(R, \{\gamma_{C_i}(k)\}_{i=1,\ldots,m,k=2,\ldots,n})$ of it, and choose a monotonically decreasing utility contest T_i that has higher rent dissipation than C_i for every i. If $T_1, \ldots, T_m$ satisfy $p_i(T_1, \ldots, T_m) \ne \tilde{p}_i$ for some i, then $W_S(T_1, \ldots, T_m) < W_S(C_1, \ldots, C_m)$, which means $(T_1, \ldots, T_m)$ does not maximize W_S any more, and $\mathcal{S}_i = \{C_i, T_i\}$ is then a counterexample.

Example D.4. When contest designers value keeping the reward differently, Theorem 5.1 no longer holds. For example, consider $m = n = 2, R_1 = R_2 = 1, \alpha_1 = 0, \alpha_2 = 0.9$. By Eq. (13), $W_S(\boldsymbol{C}) = 2 - (1 - p_1(\boldsymbol{C}))^2 - 0.1 \cdot p_1(\boldsymbol{C})^2$. Consider the contests C, T defined in Example D.2 and suppose $\mathcal{S}_1 = \mathcal{S}_2 = \{C, T\}$ (i.e., $\mathcal{S}_1$ is different than in Example D.2). Since $\mathrm{MRD}(\mathcal{S}_1) = \mathrm{MRD}(\mathcal{S}_2) = \{T\}$, Theorems 3.1 and 3.3 imply that the unique equilibrium is (T, T). Due to symmetry, $\boldsymbol{p}(T, T) = (0.5, 0.5)$, so $W_S(T, T) = 2 - 0.5^2(1 + 0.1) = 1.725$. However if contest designers choose (non-equilibrium) contests (C, T), contestants' equilibrium participation probabilities as calculated in Example D.2 are $\tilde{p}_1 = \frac{16}{31}$ and $\tilde{p}_2 = \frac{15}{31}$. So, $W_S(C, T) = 2 - (1 - \tilde{p}_1)^2 - 0.1 \cdot (\tilde{p}_1)^2 > 1.739 > W_S(T, T)$. Crucially, changing the α_i's does not change contestants' equilibrium choices, although it does change the social welfare. Therefore, in this example, equilibrium contests do not maximize the social welfare.

Appendix E. Supplementary material

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.geb.2025.07.001.

Data availability

No data was used for the research described in the article.

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