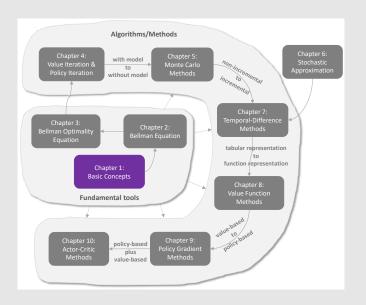
### Lecture 1: Basic Concepts

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#### Outline



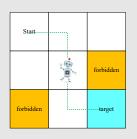
Shiyu Zhao 1/27

#### Contents

- First, introduce fundamental concepts in reinforcement learning (RL) by examples.
- Second, formalize the concepts in the context of Markov decision processes.

Shiyu Zhao 2/27

## A grid-world example





An illustrative example used throughout this course:

- Grid of cells: Accessible/forbidden/target cells, boundary.
- Very easy to understand and useful for illustration

#### Task:

- Given any starting area, find a "good" way to the target.
- How to define "good"? Avoid forbidden cells, detours, or boundary.

Shiyu Zhao 3/27

*State*: The status of the agent with respect to the environment.

• For the grid-world example, the location of the agent is the state. There are nine possible locations and hence nine states:  $s_1, s_2, \ldots, s_9$ .

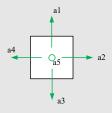
sl	s2	s3
s4	s5	s6
s7	s8	s9

State space: the set of all states  $S = \{s_i\}_{i=1}^9$ .

#### Action

*Action*: For each state, there are five possible actions:  $a_1, \ldots, a_5$ 

- $a_1$ : move upward;
- $a_2$ : move rightward;
- a<sub>3</sub>: move downward;
- a4: move leftward;
- *a*<sub>5</sub>: stay still;



sl	s2	s3
s4	s5	s6
s7	s8	s9

Action space of a state: the set of all possible actions of a state.

$$\mathcal{A}(s_i) = \{a_k\}_{k=1}^5.$$

Question: can different states have different sets of actions?



When taking an action, the agent may move from one state to another. Such a process is called *state transition*.

• Example: At state  $s_1$ , if we choose action  $a_2$ , then what is the next state?

$$s_1 \xrightarrow{a_2} s_2$$

• Example: At state  $s_1$ , if we choose action  $a_1$ , then what is the next state?

$$s_1 \xrightarrow{a_1} s_1$$

- State transition describes the interaction with the environment.
- Question: Can we define the state transition in other ways? Simulation vs physics

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Pay attention to forbidden areas: Example: at state  $s_5$ , if we choose action  $a_2$ , then what is the next state?

• Case 1: the forbidden area is accessible but with penalty. Then,

$$s_5 \xrightarrow{a_2} s_6$$

• Case 2: the forbidden area is inaccessible (e.g., surrounded by a wall)

$$s_5 \xrightarrow{a_2} s_5$$

We consider the first case, which is more general and challenging.

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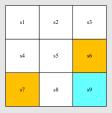


### Tabular representation: We can use a table to describe the state transition:

	$a_1$ (upward)	$a_2$ (rightward)	$a_3$ (downward)	$a_4$ (leftward)	$a_5$ (still)
$s_1$	$s_1$	$s_2$	$s_4$	$s_1$	$s_1$
$s_2$	$s_2$	$s_3$	$s_5$	$s_1$	$s_2$
$s_3$	$s_3$	$s_3$	$s_6$	$s_2$	$s_3$
$s_4$	$s_1$	$s_5$	$s_7$	$s_4$	$s_4$
$s_5$	$s_2$	$s_6$	$s_8$	$s_4$	$s_5$
$s_6$	$s_3$	$s_6$	$s_9$	$s_5$	$s_6$
87	84	<i>s</i> <sub>8</sub>	87	87	$s_7$
<i>s</i> <sub>8</sub>	$s_5$	89	<i>s</i> <sub>8</sub>	87	$s_8$
<b>s</b> 9	$s_6$	89	89	$s_8$	$s_9$

Can only represent deterministic cases.

Shiyu Zhao 8/27



State transition probability: use probability to describe state transition!

- Intuition: At state  $s_1$ , if we choose action  $a_2$ , the next state is  $s_2$ .
- Math:

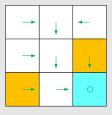
$$p(s_2|s_1, a_2) = 1$$
  
 $p(s_i|s_1, a_2) = 0 \quad \forall i \neq 2$ 

Here it is a deterministic case. The state transition could be stochastic (for example, wind gust).

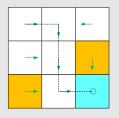
Shiyu Zhao 9/27

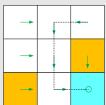
*Policy* tells the agent what actions to take at a state.

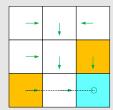
Intuitive representation: We use arrows to describe a policy.



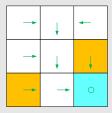
Based on this policy, we get the following trajectories with different starting points.







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Mathematical representation: using conditional probability

For example, for state  $s_1$ :

$$\pi(a_1|s_1) = 0$$

$$\pi(a_2|s_1) = 1$$

$$\pi(a_3|s_1) = 0$$

$$\pi(a_4|s_1) = 0$$

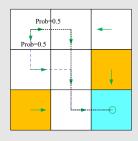
$$\pi(a_5|s_1) = 0$$

It is a deterministic policy.

Shiyu Zhao 11/2

There are stochastic policies.

For example:



In this policy, for  $s_1$ :

$$\pi(a_1|s_1)=0$$

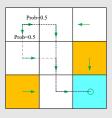
$$\pi(a_2|s_1) = 0.5$$

$$\pi(a_3|s_1) = 0.5$$

$$\pi(a_4|s_1)=0$$

$$\pi(a_5|s_1) = 0$$

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**Tabular representation** of a policy: how to use this table.

	$a_1$ (upward)	$a_2$ (rightward)	$a_3$ (downward)	$a_4$ (leftward )	$a_5$ (still)
$s_1$	0	0.5	0.5	0	0
$s_2$	0	0	1	0	0
$s_3$	0	0	0	1	0
$s_4$	0	1	0	0	0
$s_5$	0	0	1	0	0
$s_6$	0	0	1	0	0
87	0	1	0	0	0
$s_8$	0	1	0	0	0
<b>s</b> 9	0	0	0	0	1

Can represent either deterministic or stochastic cases.

#### Reward is one of the most unique concepts of RL.

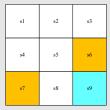
Reward: a real number we get after taking an action.

- A positive reward represents encouragement to take such actions.
- A negative reward represents punishment to take such actions.

#### Questions:

- Can positive indicate punishment and negative indicate encouragement?
  - Yes.
  - In this case, reward may called cost.
- What about a zero reward?
  - Relative values matter, not absolute values.
  - $r = \{+1, -1\}$  becomes  $r = \{+2, 0\}$  will not change the optimal policy.

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In the grid-world example, the rewards are designed as follows:

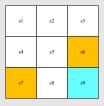
- ullet If the agent attempts to get out of the boundary, let  $r_{
  m bound}=-1$
- ullet If the agent attempts to enter a forbidden cell, let  $r_{
  m forbid}=-1$
- ullet If the agent reaches the target cell, let  $r_{
  m target}=+1$
- ullet Otherwise, the agent gets a reward of r=0.

Reward can be interpreted as a **human-machine interface**, with which we can guide the agent to behave as what we expect.

For example, with the above designed rewards, the agent will try to avoid getting out of the boundary or stepping into the forbidden cells.

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#### Reward



#### **Tabular representation** of *reward transition*: how to use the table?

	$a_1$ (upward)	$a_2$ (rightward)	$a_3$ (downward)	$a_4$ (leftward )	$a_5$ (still)
$s_1$	$r_{ m bound}$	0	0	$r_{ m bound}$	0
$s_2$	$r_{ m bound}$	0	0	0	0
$s_3$	$r_{ m bound}$	$r_{ m bound}$	$r_{ m forbid}$	0	0
$s_4$	0	0	$r_{ m forbid}$	$r_{ m bound}$	0
$s_5$	0	$r_{ m forbid}$	0	0	0
<i>s</i> <sub>6</sub>	0	$r_{ m bound}$	$r_{\mathrm{target}}$	0	$r_{ m forbid}$
$s_7$	0	0	$r_{ m bound}$	$r_{ m bound}$	$r_{ m forbid}$
$s_8$	0	$r_{ m target}$	$r_{ m bound}$	$r_{ m forbid}$	0
$s_9$	$r_{ m forbid}$	$r_{ m bound}$	$r_{ m bound}$	0	$r_{ m target}$

Can only represent deterministic cases.

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#### Mathematical description: conditional probability

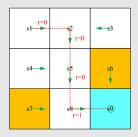
- Intuition: At state  $s_1$ , if we choose action  $a_1$ , the reward is -1.
- Math:  $p(r = -1|s_1, a_1) = 1$  and  $p(r \neq -1|s_1, a_1) = 0$

#### Remarks:

Here it is a deterministic case. The reward transition could be stochastic.
 For example, if you study hard, you will get rewards. But how much is uncertain.

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## Trajectory and return



A trajectory is a state-action-reward chain:

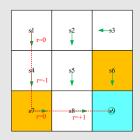
$$s_1 \xrightarrow[r=0]{a_2} s_2 \xrightarrow[r=0]{a_3} s_5 \xrightarrow[r=0]{a_3} s_8 \xrightarrow[r=1]{a_2} s_9$$

The *return* of this trajectory is the sum of all the rewards collected along the trajectory:

return = 
$$0 + 0 + 0 + 1 = 1$$

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# Trajectory and return



A different policy gives a different trajectory:

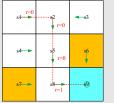
$$S_1 \xrightarrow[r=0]{a_3} S_4 \xrightarrow[r=-1]{a_3} S_7 \xrightarrow[r=0]{a_2} S_8 \xrightarrow[r=+1]{a_2} S_9$$

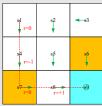
The return of this path is:

return = 
$$0 - 1 + 0 + 1 = 0$$

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### Trajectory and return



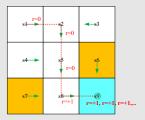


#### Which policy is better?

- Intuition: the first is better, because it avoids the forbidden areas.
- Mathematics: the first one is better, since it has a greater return!
- Return could be used to evaluate whether a policy is good or not (see details in the next lecture)!

Shiyu Zhao 20 / 27

#### Discounted return



A trajectory may be infinite:

$$s_1 \xrightarrow{a_2} s_2 \xrightarrow{a_3} s_5 \xrightarrow{a_3} s_8 \xrightarrow{a_2} s_9 \xrightarrow{a_5} s_9 \xrightarrow{a_5} s_9 \dots$$

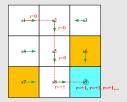
The return is

return = 
$$0 + 0 + 0 + 1 + 1 + 1 + \dots = \infty$$

The definition is invalid since the return diverges!

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#### Discounted return



Need to introduce a *discount rate*  $\gamma \in (0,1)$ :

discounted return 
$$= 0 + \gamma 0 + \gamma^2 0 + \gamma^3 1 + \gamma^4 1 + \gamma^5 1 + \dots$$
$$= \gamma^3 (1 + \gamma + \gamma^2 + \dots) = \gamma^3 \frac{1}{1 - \gamma}.$$

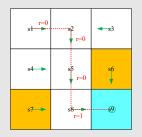
Roles: 1) the sum becomes finite; 2) balance the far and near future rewards:

- ullet If  $\gamma$  is close to 0, the value of the discounted return is dominated by the rewards obtained in the near future.
- ullet If  $\gamma$  is close to 1, the value of the discounted return is dominated by the rewards obtained in the far future.

Shiyu Zhao 22/2

### **Episode**

When interacting with the environment following a policy, the agent may stop at some *terminal states*. The resulting trajectory is called an *episode* (or a trial).



Example: episode

$$s_1 \xrightarrow[r=0]{a_2} s_2 \xrightarrow[r=0]{a_3} s_5 \xrightarrow[r=0]{a_3} s_8 \xrightarrow[r=1]{a_2} s_9$$

An episode is usually assumed to be a finite trajectory. Tasks with episodes are called *episodic tasks*.

Shiyu Zhao 23/2

### **Episode**

Some tasks may have no terminal states, meaning the interaction with the environment will never end. Such tasks are called *continuing tasks*.

In the grid-world example, should we stop after arriving the target?

- ullet Treat the target state as a normal state with a policy. The agent can still leave the target state and gain r=+1 when entering the target state.
- We don't need to distinguish the target state from the others and can treat it as a normal state.

Shiyu Zhao 24 / 27

# Markov decision process (MDP)

#### Key elements of MDP:

- Sets:
  - ullet State: the set of states  ${\cal S}$
  - Action: the set of actions A(s) is associated for state  $s \in S$ .
  - Reward: the set of rewards  $\mathcal{R}(s,a)$ .
- Probability distribution (or called system model):
  - State transition probability: at state s, taking action a, the probability to transit to state s' is p(s'|s,a)
  - Reward probability: at state s, taking action a, the probability to get reward r is p(r|s,a)
- Policy: at state s, the probability to choose action a is  $\pi(a|s)$
- Markov property: memoryless property

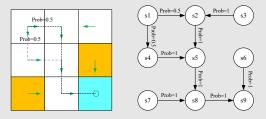
$$p(s_{t+1}|a_t, s_t, \dots, a_0, s_0) = p(s_{t+1}|a_t, s_t),$$
  
$$p(r_{t+1}|a_t, s_t, \dots, a_0, s_0) = p(r_{t+1}|a_t, s_t).$$

All the concepts introduced in this lecture can be put in the framework in MDP.

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## Markov decision process (MDP)

The grid world could be abstracted as a more general model, *Markov process*.



The circles represent states and the links with arrows represent the state transition.

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### Summary

By using grid-world examples, we demonstrated the following key concepts:

- State
- Action
- State transition, state transition probability p(s'|s,a)
- Reward, reward probability p(r|s, a)
- Trajectory, episode, return, discounted return
- Markov decision process

Shiyu Zhao 27 / 27